(Preliminary Version)

Threat Misestimations and the Role of NGOs in International Risk Management

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Abstract

In this paper, we investigate the impacts of misestimation concerning the severity of a threat on the provision of risk-management public goods in an alliance. Using the alliance model developed by Ihori et al. (2014), we examine how misestimation of a threat affects burden-sharing among allies. The ally may "over"-estimate the threat in the sense that the estimated level of threat is higher than the true level. We will show that if an ally overestimates the severity of threat, it may contribute more to the public goods than it does when it estimates the precise severity.

Then, we investigate the influence of interest groups on threat estimation in an alliance wherein two countries voluntarily provide a self-protection public good to reduce the risk of a bad event. We hypothetically assume the existence of an international NGO (Non-Governmental Organization) which can affect governments' estimations through lobbying. If the unit cost of lobbying satisfies a condition, the NGO will effectively choose the socially optimal level of lobbying, which leads allied countries to choose the socially optimal level of a public good at a (non-cooperative) Nash equilibrium. Our analysis sheds new light on the normative role of the NGOs. Intentionally or unintentionally, international NGOs may pessimistically bias the government's estimation of loss in the bad state. This then increases the equilibrium level of the public good and improves the welfare of the allies.

Keywords: international public goods, self-protection, self-insurance, misestimation.

JEL codes: D81, F50, H41.

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1: Introduction

In this paper, we investigate the impacts of misestimation concerning the severity of a threat on collective risk management. In general, it is significantly difficult for a policymaker or government of a country in the real world to estimate the precise severity of a threat for the country. Although the government's agency intends to collect precise information on this severity, the collected information could contain significant errors. It is also often biased in the bureaucratic process of the administration. Moreover, interest groups try to influence the evaluation of the information and the policymaking process through lobbying.

Using the alliance model developed by Ihori et al. (2014), we examine how misestimation of a threat affects burden-sharing among allies. The ally may "over"-estimate the threat in the sense that the estimated level of threat is higher than the true level of the threat. We will show that if an ally overestimates the severity of threat to the alliance, it may contribute more to the public goods than it does when it estimates the precise severity. Although the threat is "over"-estimated in the sense that the estimated level is higher than the true one, the welfare of the allies resulting from this estimation may be more efficient than the correct estimation because the free-riding in the voluntary provision of public goods could be alleviated so as to attain the socially optimal level. The prefix OVER of our term "overestimation" merely means that the estimated value is higher than the true parameter.

Namely, we explore the possibility that overestimation (or more generally misestimation) of the threat may improve the welfare of the allies. We argue that there may exist a socially optimal level of overestimation of threat, which would induce governments to voluntarily provide the socially optimal amount of a self-protection public good in the Nash equilibrium. Although it is somewhat contradictory terminology, "socially optimal overestimation" means that the higher estimation than the true one could result in the socially optimal provision of the public good in the second best economy where private provision of the public good is under-provided.

Extant literature on the voluntary provision of public goods reports that ambiguity in the contribution by other players mitigates the free-riding effect. Eichberger and Kelsey (2002) investigated the relationship between externality and ambiguity. They defined ambiguity as follows: "Ambiguity refers to situations in which individuals have to make decisions when the relevant probabilities are unknown (Eichberger and Kelsey 2002, p.437)." [1] They showed that the amount of a public good voluntarily provided by players increases with ambiguity. Bailey, Eichberger, and Kelsey (2005) showed that if ambiguity concerning the contribution by other players persists, the

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[1] For the details of the definition of ambiguity, please refer to Eichberger and Kelsey (2002).

provision of public good increases with the population of the economy. Following the conventional literature on the private provision of public goods, these two pieces of research assumed that the provision of a public good is given as the sum of contributions. Unlike them, Kelsey and le Roux (2017) constructed a two-player game in which players voluntarily contribute to the provision of a public good and the benefit of the public good is given by best-shot or weakest-link technologies.[2] They suppose that players are ambiguity-averse and show that if the public good is provided with a best-shot technology, high ambiguity concerning the other player's contribution causes both players to contribute the highest level to the public good and that under a weakest-link technology, high ambiguity causes them to contribute the lowest level to the public good.[3] Overall, in the context of collective risk management in an alliance, previous research suggests that, if allies are uncertain about the security expenditures of other allies, they contribute more to the security of the alliance.

However, that allies are uncertain mainly about the security expenditures of other allies is not a straightforward assumption. It seems more plausible to assume that allies are uncertain about the threat to their security more than others' security expenditures. We, therefore, prioritize the uncertainty of the threat magnitude itself and investigate a situation wherein allies may over/underestimate the threat to their alliance. But for the sake of simplicity, we make the contrary assumption that each ally is certain about the contributions made by other allies.

We formulate misestimations concerning the threat as a bias in the estimate of loss in a bad event. In models without misestimations, security spending affects the welfare of allies through two channels. First, a preventive force of security spending affects loss in a bad state. For example, preventive military power could reduce damage to the alliance in case of war. Second, bellicosity affects the probability of a bad state. When security spending is more provocative, a bad state will more likely occur. Among these two channels, it is more straightforward to investigate the analytical implications of over/underestimating a loss. Moreover, the analytical implications of the second channel are more complicated. We, therefore, concentrate on the first issue and incorporate misestimations about the magnitude of loss in the bad state.

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[2] Hirshleifer (1983) introduced the concept of the best-shot and weakest-link technologies. Under a best-shot technology, the provision of the public good is equal to the maximum of contributions made by any single player. Under a weakest-link technology this provision equals the minimum of individual contributions.

[3] Though Kelsey and le Roux (2017) mathematically defined "ambiguity-averse," Eichberger and Kelsey (2014) intuitively explain the meaning of "ambiguity-averse" as follows: if individuals are ambiguity-averse, "they would pay some positive amount of money to avoid a situation where probabilities are poorly defined (Eichberger and Kelsey 2014, p.484)."

Because of such threat misestimations, countries may overprovide or underprovide public goods for international security. For example, consider the Iraq War. Before the breakout of the war, the US insisted that Saddam Hussein was hiding weapons of mass destruction. However, the US could not find the weapons after its invasion of Iraq. If Saddam had owned such weapons and *was* connected to terrorists, his potential threat could have been devastating; in short, the US government overestimated Saddam's threat. We can also find other examples of the overestimation of an opponent's threat, such as the Cold War era's "Missile Gap" in the late 1950s. On the other hand, examples of threat underestimation in our history could include a well-known fact: in the 1930s, the UK and other European countries underestimated a remilitarized Germany under Adolf Hitler.

The first half of this paper investigates how misestimations concerning a threat affect burden-sharing in an alliance. In section 2, we construct a two-country model of collective risk management, wherein we introduce the overestimation of the magnitude of loss in the bad state. We assume that both countries face a common risk of emergency. A bad event stochastically happens. If the bad event happens, both countries suffers from a loss in the event. To manage this risk, they voluntarily provide two public goods. One is a self-protection public good, which reduces the probability of the bad event. The other is a self-insurance public good, which decreases the loss in the bad event. We then show that if two allies overestimate their loss in a bad state, if they contribute to both public goods in the Nash equilibrium and if the estimate of one ally deviates more from the true loss than that of the other, the former ally consumes more than the latter in the bad state. Moreover, we show that difference in total security expenditures between allies is not influenced by misestimations but that the compositions of their security expenditures are influenced. If a high-income ally overestimates a loss compared with a low-income ally, the former contributes less to self-protection and more to self-insurance than the latter. Then, the difference in self-protection (or in self-insurance) between the two allies becomes smaller (larger) than it is in the absence of overestimation.

In the second half of this paper, we conduct a "normative analysis of misestimation." Our question is: will misestimations maximize the utilitarian social welfare of individual allies? In a one-country model, misestimations can "hinder" a government from achieving an optimal supply of security. However, in an alliance model, voluntary provision of a public good results in under-provision of the public good (under a non-cooperative Nash solution) because the nature of a public good itself presents a free-riding problem. Thus, a threat overestimation could stimulate public good provision and mitigate the free-riding problem.

In section 3 we simplify our investigation by considering a single-public-good model wherein allies voluntarily provide a self-protection public good only. We discuss whether threat overestimation can induce countries to provide a socially optimal amount of public good in total.

In section 4 we investigate the influence of interest groups on threat estimation. Although various interest groups may affect the government in the process of threat estimation, we hypothetically assume the existence of an international NGO (Non-Governmental Organization) located outside the alliance which can affect governments' behaviors when they estimate losses. The reason we focus on international NGOs outside the alliance is twofold: one is their increasing presence in the international policy arena, which we discuss below, and the other is analytical simplicity. If we assume that an agent inside the alliance can influence the estimation of loss, the welfare analysis should also consider the payoff of the agent, which would require a much more complicated analysis than ours.

We investigate whether this NGO could effectively choose "optimal" level of bias such that the allies provide a socially optimal amount of public good in Nash equilibrium. For example, an international NGO that protects the global environment would have an incentive to induce allies' policymakers to pessimistically estimate environmental emergency costs. If policymakers actually estimate the cost of emergency to be higher than the true cost, they may choose a more stringent environmental policy than they do in the absence of overestimation.

In the realm of international affairs, NGOs are, in fact, providers of international public goods. For decades, they have contributed to various public goods, such as education, public healthcare, and international development assistance.[4] They have also indirectly stimulated international public goods by persuading governments to provide more of them. For example, they participate in international conferences and facilitate discourse in line with their objectives. In the domain of international security, some NGOs have played an essential role in developing arms control treaties, such as the Convention on the Prohibition of the Use, Stockpiling, Production and Transfer of Anti-Personnel Mines and on their Destruction, which is known as the *Ottawa Treaty*. However, their participation in international security has been relatively limited compared with developing assistance and public health. In this section, we shed light on the normative role of NGOs in the latter case. That is, we focus on the normative role of NGOs as agents improving governments' decision-making in non-cooperative Nash equilibrium when providing somewhat biased information. We use the term "NGO" as organizations whose objective is to solve global issues, such as climate change or a pandemic of infectious diseases.

The assumptions and results of our analysis can be summarized as follows. We consider an alliance of two countries sharing a self-protection public good. An NGO located outside the alliance,

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[4] Besley and Ghatak (2001, 2017) theoretically analyzed the relationship between a government and an NGO in the provision of public goods.

we assume, aims to bias government estimates of loss in those governments' decision-making process. Our NGO measures its benefit from self-protection achieved and pays the cost of lobbying for overestimated losses in bad states; examples may include political organizations comprising stringent environmentalists. We assume that the governments of the allied countries determine their contribution to the public good after the NGO determines its level of lobbying. We investigate the properties of the best-response function of the allied countries and conduct several numerical simulations. Our simulation results, presented in appendix C of this paper, show that, depending on the cost of lobbying, the NGO can effectively choose the socially optimal level of lobbying, which leads allied countries to choose the socially optimal level of self-protection at (non-cooperative) Nash equilibrium.

In a conventional model of voluntary provision of public goods, each player does not consider the external benefit of its provision enjoyed by other players. All players together, therefore, provide an inefficiently small amount of public good. If a planner supervises the players of the voluntary provision game, collects tax from them, and pays a matching grant for the provision of public goods, he/she can induce players to provide the socially optimal amount. However, we cannot utilize this policy instrument in the case of international risk management because no "World Government" exists that can compulsorily collect a tax from allied countries. In place of such "World Government," international NGOs (intentionally or unintentionally) might pessimistically bias the government's estimation of the loss in a bad state, thereby increasing the equilibrium level of self-protection, and improve the welfare of all allies.

This paper consists of five sections. In section 2, we construct the basic model of overestimation of loss in the bad state. This allows us to investigate how bias in threat estimations affect burden-sharing among allies. Generally, overestimation of one variable (loss in a bad state) alone may not induce a government to allocate socially optimal levels of two variables (self-insurance and self-protection). In section 3, we thus simplify the multi-variable model by omitting self-insurance and investigate whether overestimation could generate a socially optimal amount of the self-protection. With this simplification, we will show that choosing an appropriate level of overestimation may induce countries to choose a socially optimal level of self-protection. In section 4, we extend our model to introduce a hypothetical international NGO's optimizing behavior. We assume that it may effectively influence the level of mis-estimate and that its objective is not to maximize the welfare of the allied countries but to maximize its own payoff. Then, we discuss the possibility that allied countries actually choose socially optimal levels of public good in the Nash equilibrium, owing to NGOs. Section 5 concludes this paper.

2: Exogenous Misestimation and Exploitation Hypothesis

Although each government maximizes the expected welfare of its representative household, a government's estimation of loss could be influenced by external organizations, such as NGOs and lobbyist groups. For example, environmental protectionists might extremely emphasize the threat of climate change, while arms-producing companies may emphasize the threat of terrorist attacks. In the following sections, we incorporate overestimation (or underestimation) of the magnitude of loss in a bad state, and then investigate how the misestimation influences allies' burden-sharing in risk management.

2.1. Analytical Framework

In the following, we construct a two-country model wherein country A and B constitute an alliance. Mainly we describe the maximization problem for the government of country A only. We could easily describe a similar maximizing problem for country B by replacing the superscript A with B. We assume that there are two contingent states of the world: a good state ("1") and a bad state ("0"). If the state is bad, the income of each country is partially lost.

With misestimation, the government of country A maximizes its welfare based on biased information of loss in the bad state. Hence, the objective of the maximization is the estimated expected welfare of country A. We here use the term "estimated" in the sense that country A does not know the true loss in the bad state but estimates it when deciding its budget allocation. We assume that both the true loss and the estimated loss are exogenously given. The estimated expected welfare of country A is then given by the following:

$$\tilde{W}^{A} = pU(\tilde{C}^{1A}) + (1-p)U(\tilde{C}^{0A}), \tag{1}$$

where $\tilde{W}^{A}$ is country A's estimated expected utility, $\tilde{C}^{1A}$ is country A's estimated consumption if the state of the world is good, $\tilde{C}^{0A}$ is A's estimated consumption if the state is bad, $U(.)$ is a utility function, and p is the probability of the good state. We denote the value of $U(.)$ as follows:

$$\tilde{U}^{iA} \equiv U(\tilde{C}^{iA}) \quad \text{for} \quad i = 0,1.$$

We assume that the utility function, $U(.)$, is increasing and concave:[5]

$$\tilde{U}_{Y}^{iA} \equiv dU(\tilde{C}^{iA}) / d\tilde{C}^{iA} > 0, U_{YY}^{iA} \equiv d^{2}\tilde{U}(\tilde{C}^{iA}) / d(\tilde{C}^{iA})^{2} < 0, (i = 0,1). \tag{2}$$

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[5] In the notation of the derivative of the utility function, we use subscript Y. As defined below, Y represents the unconditional income of the country. The reason why we use Y instead of C is that we focus on the income effect throughout this paper.

We also assume the utility function satisfies the Inada condition:

$$\lim_{C \to 0} dU(C)/dC = +\infty \quad \text{and} \quad \lim_{C \to +\infty} dU(C)/dC = 0\,. \tag{3}$$

The probability of the good state, p, increases with the provision of self-protection. Self-protection is an international public good. The self-protection purchased by both countries raises the probability of the good state with that benefit of increasing probability of the good state being nonrival and nonexcludable. Its amount is given by the sum of the contributions of allied countries as follows:

$$M_1 = m_1^A + m_1^B, \tag{4}$$

where M_1 is the total amount of the self-protection public good, m_1^A is country A's contribution to the public good, and m_1^B is country B's contribution. The probability of the good state is given by:

$$p = p(M_1), \tag{5}$$

where

$$p''(M_1) > 0,\, p''(M_1) < 0,\, p(M_1) \in (0,1) \quad \text{for any} \quad M_1 \geq 0\,. \tag{6}$$

We assume that countries A and B can purchase self-insurance, which reduces a loss in the bad state and that the self-insurance is an international public good in this alliance. When government of country A determines its self-insurance and self-protection expenditures, its budget constraint is given as follows:

$$\tilde{C}^{1A} = Y^A - m_1^A - m_2^A; \tag{7}$$

$$\tilde{C}^{0A} = Y^A - \tilde{L}^A - m_1^A + \mathrm{L}(s)\,, \tag{8}$$

where Y^A is country A's income, $\tilde{L}^A$ is its estimated loss in the bad event, m_2^A, its contribution to the self-insurance public good, $\mathrm{L}(s)$ is the self-insurance benefit in the bad state, and s is the self-insurance input in the bad state. The estimated loss may be formulated as the sum of the true loss and the bias in estimation:

$$\tilde{L}^A = \bar{L}^A + \alpha^A, \tag{9}$$

where $\bar{L}^A$ is the true loss in the bad state and α^A measures the overestimation of that loss (which is given exogenously in this section). If A underestimates its loss, we obtain $\alpha^A < 0$. The government of A presupposes $\tilde{L}^A$ as the true estimated loss when it chooses its contributions to the public good.

The self-insurance benefit function $\mathrm{L}(s)$ is increasing and concave. We assume that the marginal product or benefit of self-insurance is not higher than unity. Thus, we have

$$\mathrm{L}' \equiv \frac{d\,\mathrm{L}}{ds} \in (0,1], \mathrm{L}'' \equiv \frac{d^2\,\mathrm{L}}{ds^2} \in (-\infty, 0]. \tag{10}$$

Self-insurance input, s, is the total self-insurance premium divided by the price of self-insurance. As in Ihori et al.(2014), we assume that the price of self-insurance is actuarially fair. Then, we have

$$s = \frac{M_2}{\left(\frac{1-p}{p}\right)} = \frac{pM_2}{1-p}, \tag{11}$$

where $M_2 = m_2^A + m_2^B$ is the total payment of the self-insurance premiums paid by both countries and the denominator of the right-hand side (RHS) of (11), $(1-p)/p$, is the actuarially fair price of self-insurance.

The time structure of this model is as follows. Both A and B simultaneously determine the allocations of their endowments to self-insurance and self-protection. Then, the state of the world is stochastically determined. If the state is bad, the true loss is revealed. The *ex post* budget constraints of country A are thus given by the following:

$$C^{1A} = Y^A - m_1^A - m_2^A; \tag{12}$$

$$C^{0A} = Y^A - \bar{L}^A - m_1^A + \mathrm{L}(s), \tag{13}$$

where C^{1A} is A's consumption in the good state, C^{0A} is consumption in the bad state, and $\bar{L}^A$ is country A's true actual (or ex post) loss in the bad state.[6]

2.2. Individual Optimization and Nash Equilibrium

In this subsection, we derive the conditions for individual optimization and define "Nash equilibrium with misestimation." The government of country A solves the following maximization problem:

$$\max_{m_1^A, m_2^A} \tilde{W}^A = pU(\tilde{C}^{1A}) + (1-p)U(\tilde{C}^{0A})$$

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6 The value of C^{1A} is identical to that of $\tilde{C}^{1A}$. We distinguish $\tilde{C}^{1A}$ from C^{1A} to clarify that $\tilde{C}^{1A}$ represents the consumption in a good state estimated in the first period, while C^{1A} is the realized consumption in the second state.

subject to Eqs. (4), (5), (7), (8) and (11).

The first order condition for interior self-protection is given as follows:

$$\frac{\partial \tilde{W}^{A}}{\partial m_1^A} = p'\left(\tilde{U}^{1A} - \tilde{U}^{0A}\right) - \left\{ p\tilde{U}_Y^{1A} + (1-p)\left(1 - \frac{\partial \mathrm{L}}{\partial M_1}\right)\tilde{U}_Y^{0A} \right\} = 0 . \tag{14}$$

The first order condition for an interior self-insurance is given as follows:

$$\frac{\partial \tilde{W}^{A}}{\partial m_2^A} = p\left(\mathrm{L}'\tilde{U}_Y^{0A} - \tilde{U}_Y^{1A}\right) = 0 . \tag{15}$$

We also assume that the second order conditions for the expected welfare maximization are satisfied:

$$\frac{\partial^2 \tilde{W}^{A}}{\partial (m_1^A)^2} < 0, \quad \frac{\partial^2 \tilde{W}^{A}}{\partial (m_2^A)^2} < 0 \quad \text{and} \quad \frac{\partial^2 \tilde{W}^{A}}{\partial^2 m_1^A}\frac{\partial^2 \tilde{W}^{A}}{\partial^2 m_2^A} - \left(\frac{\partial^2 \tilde{W}^{A}}{\partial m_1^A \partial m_2^A}\right)^2 > 0 . \tag{16}$$

Then, country A's best response function is given as follows:

$$m_1^A = m_1^A(m_1^B, m_2^B, Y^A, \tilde{L}^A) , \tag{17}$$

$$m_2^A = m_2^A(m_1^B, m_2^B, Y^A, \tilde{L}^A) . \tag{18}$$

The Nash equilibrium of this model, $(m_1^{A*}, m_2^{A*}, m_1^{B*}, m_2^{B*})$, is defined as the solution to the following system:

$$m_1^{A*} = m_1^A(m_1^{B*}, m_2^{B*}, Y^A, \tilde{L}^A) ,$$

$$m_1^{B*} = m_1^B(m_1^{B*}, m_2^{B*}, Y^A, \tilde{L}^A) ,$$

$$m_1^{B*} = m_1^B(m_1^{A*}, m_2^{A*}, Y^B, \tilde{L}^B) ,$$

$$m_2^{B*} = m_1^B(m_1^{A*}, m_2^{A*}, Y^B, \tilde{L}^B) .$$

When country A maximizes its expected welfare, it estimates its consumption in both states. The estimated consumptions in the good and bad states under Nash equilibrium allocations to self-insurance and self-protection are denoted $\tilde{C}^{1A*}$ and $\tilde{C}^{0A*}$, respectively. They are given as follows:

$$\tilde{C}^{1A*} = Y^{A*} - m_1^{A*} - m_2^{A*}, \tag{19}$$

$$\tilde{C}^{0A*} = Y^A - \bar{L}^A - \alpha^A - m_1^{A*} + \mathrm{L}(s)\,. \tag{20}$$

The true (or *ex post*) consumptions in good and bad states under the Nash equilibrium are defined as C^{1A*} and C^{0A*}, and are given as follows:

$$C^{1A*} = Y^{A*} - m_1^{A*} - m_2^{A*},$$

$$C^{0A*} = Y^A - \bar{L}^A - m_1^{A*} + \mathrm{L}(s)\,.$$

We refer to the probability-weighted realized utility as the true expected welfare:

$$W^{A*} = p(M_1^*)U(C^{1A*}) + (1 - p(M_1^*))U(C^{0A*}), \tag{21}$$

where C^{1A*} and C^{0A*} are the Nash equilibrium levels of consumptions in good and bad states, respectively.

2.3. Exploitation Hypothesis in Misestimation Model

Concentrating on an interior Nash equilibrium, we examine the impact of misestimations on alliance burden-sharing. We show that, even if there is misestimation, both countries consume the same amount in both states of the world from the estimated *ex ante* viewpoint. However, now misestimations affect real *ex post* consumption in the bad state. Then, the true expected welfare is not identical between allies. We also show that bias in estimation do not affect the inter-ally difference in total security expenditure but do affect the resource allocation between self-insurance and self-protection. These results are summarized in the following propositions. The proof of this proposition is given in appendix A.

Proposition 1 *Suppose that all elements of the Nash equilibrium vector of contributions,* $(m_1^{A*}, m_2^{A*}, m_1^{B*}, m_2^{B*})$*, are positive and that*

$$\tilde{R}^{0A}\,\mathrm{L}' \geq \tilde{R}^{1A} \text{ and } \tilde{R}^{0B}\,\mathrm{L}' \geq \tilde{R}^{1B}, \tag{22}$$

where $\tilde{R}^i \equiv -U_{YY}(\tilde{C}^i)/U_Y(\tilde{C}^i)$ *for* $i = 0A, 1A, 0B, 1B$ *is absolute risk aversion while estimated consumption is* $\tilde{C}^i$*. Then, estimated consumptions in Nash equilibrium satisfy*

$$\tilde{C}^{0A*} = \tilde{C}^{0B*} \text{ and } \tilde{C}^{1A*} = \tilde{C}^{1B*}\,. \tag{23}$$

According to Proposition 1, if both countries contribute to both public goods and if the absolute risk aversions of both countries are sufficiently decreasing with consumption, then estimated consumptions become identical regardless of the difference in their national incomes.

However, the true expected welfare is not identical between allies as shown in the following proposition. The proof is of this proposition is given in appendix A.

Proposition 2 *Suppose that Eq. (22) is satisfied. Suppose also that the Nash equilibrium is an interior solution. Then, we have the following:*

$$C^{1A*} = C^{1B*}, \tag{24}$$

$$C^{0A*} - C^{0B*} = \alpha^A - \alpha^B, \tag{25}$$

$$W^{A*} > W^{B*} \quad \textit{if and only if} \quad \alpha^A > \alpha^B. \tag{26}$$

According to Proposition 2, if Nash equilibrium is interior, the country with the larger overestimation bias consumes more in the bad state than the other country. This implies that the country with larger overestimation enjoys a higher true expected welfare. The intuition behind this proposition is simple. As shown in Proposition 1, if both countries contribute to both public goods, their estimated consumptions in the bad state will be identical. But the realized consumption in the bad state is larger than the estimated consumption by the amount of the overestimated of the loss. Thus, the country that makes the bigger mistake and the larger overestimate consumes more in the bad state than the other country. But in the good state, their estimated consumptions are identical. Hence, their realized consumptions in the good state are also identical because the realized consumption and estimated consumption in the good state are the same. To summarize, a country with the larger bias in estimated loss consumes more than the other country in the bad state, consumes the same as the other country in the good state, and enjoys a higher true expected welfare than the other country.

Proposition 1 also has implications for burden-sharing between the two allies, as summarized in the following proposition.

Proposition 3 *Suppose that Eq. (22) is satisfied. Suppose also that the Nash equilibrium to be an interior solution. Then, we have the following:*

$$(m_1^{A*} + m_2^{A*}) - (m_1^{B*} + m_2^{B*}) = Y^A - Y^B, \tag{27}$$

$$m_1^{A*} - m_1^{B*} = (Y^A - \bar{L}^A) - (Y^B - \bar{L}^B) + \alpha^B - \alpha^A \quad , \tag{28}$$

$$m_2^{A*} - m_2^{B*} = \bar{L}^A - \bar{L}^B + \alpha^A - \alpha^B. \tag{29}$$

Proof: The estimated consumptions of country B in good and bad states are given as follows:

$$\tilde{C}^{1B*} = Y^{B*} - m_1^{B*} - m_2^{B*}, \tag{30}$$

$$\tilde{C}^{0A*} = Y^{B} - \bar{L}^{B} - \alpha^{B} - m_1^{B*} + \mathrm{L}(s). \tag{31}$$

Substituting Eqs. (19), (20), (30) and (31) in Eq. (23), we obtain Eqs. (27) and (28). Combining Eq.(27) with Eq. (28), we obtain Eq. (29).

Proposition 3 claims that the difference in total security expenditure between allies is not influenced by biases in estimations. As shown in Eq. (27), the difference between $m_1^{A*} + m_2^{A*}$ and $m_1^{B*} + m_2^{B*}$ depends only on the difference in national income between allies. The biases in estimations of loss, α^A and α^B, affect the composition of security expenditure. For example, let us suppose that country A overestimates its loss in the bad state more than country B, $\alpha^A > \alpha^B$. This causes m_2^{A*} to be more than m_2^{B*}, while it causes m_1^{A*} to be less than m_1^{B*}. Then, the difference in self-protection contributions is smaller, while that in self-insurance contributions is larger than in the absence of misestimations. The shrinking of the gap in self-protection expenditure is canceled out by the widening of the gap in the self-insurance expenditure. But differences between countries in their equilibrium levels of total security expenditures are unaffected by the misestimations.

2.4. Numerical Examples

In this section, we conduct several numerical simulations to investigate how an overestimation of loss affects burden-sharing in the alliance. We follow our numerical simulation in Ihori et al. (2014) in the specification of the utility function and that of the probability function. The values of the parameters, which are summarized in Table 1. Both countries are endowed with identical 50 units of income and face the identical 10 units of loss in the bad state.

Table 1 Parameter values

p_0	p_e	ϕ	θ
0.25	1	0.9	0.9

Source: Authors.

Table 2 Numerical simulations of exogenous overestimation and burden-sharing

Scenario	1 Baseline	2 Overestimation by A	3 Overestimation by both
Y^A	50	50	50
Y^B	50	50	50
$\bar{L}^A$	10	10	10
$\bar{L}^B$	10	10	10
α^A	0	0.1	0.1
α^B	0	0	0.1
m_1^A	0.961	0.920	0.978
m_2^A	1.734	1.796	1.759
m_1^B	0.961	1.020	0.978
m_2^B	1.734	1.696	1.759
M_1	1.923	1.939	1.956
M_2	3.467	3.493	3.518
$m^A (= m_1^A + m_2^A)$	2.695	2.716	2.737
$m^B (= m_1^B + m_2^B)$	2.695	2.716	2.737
$\tilde{W}^A$	14.619	14.619	14.618
$\tilde{W}^B$	14.619	14.619	14.618
W^A	14.619	14.621	14.620
W^B	14.619	14.619	14.620
$m_1^A - m_1^B$	0.000	-0.100	0.000
$m_2^A - m_2^B$	0.000	0.100	0.000
$m^A - m^B$	0.000	0.000	0.000

Source: Authors.

Table 2 summarizes the results of three numerical simulations. Column 1 reports the result of a baseline scenario wherein no misestimations of loss occur. Then, both countries contribute to the self-protection and self-insurance public goods. Country A's contribution of self-insurance and self-protection are equal to those made by country B.

Column 2 reports the result of simulation wherein only country A overestimates the loss by 0.1 unit. Compared with column 1, country A increases its self-insurance contribution and decreases its self-protection contribution. On the contrary, country B decreases its self-insurance and increases its self-protection. As shown in Proposition 3, country A's self-protection is smaller than country B's by 0.1 unit, and country A's self-insurance is larger than country B's by 0.1 unit. The total security expenditure of country A is identical to that of country B. Although differences in total security expenditure are not affected by the overestimation, total security expenditure itself increased with the overestimation. In column 1, total security expenditure of country A is 2.695, while that in column 2 is 2.716. As predicted in Proposition 2, the true welfare of country A is higher than that of country B in column 2.

Column 3 illustrates the result of the simulation wherein both countries A and B overestimate the loss by 0.1 unit. Then, both countries increase their contributions to both public goods. Each country contributes more to self-insurance and self-protection than they do in the baseline. Furthermore, this overestimation of the loss is Pareto-improving, that is, the true expected welfares of both countries improve. This result highlights a normative role of misestimation.

However, it should be stressed that overestimation of the loss alone cannot generally achieve the social optimal allocation of self-insurance and self-protection because we cannot choose the optimal allocation of two endogenous variables (self-insurance and self-protection) using one exogenous variable (over/under estimation of loss). In the next section, we simplify our model to a model with a self-protection public good only and conduct a normative analysis of misestimations to attain the social optimum in the Nash equilibrium.

3: A Normative Analysis of Misestimation in Identical Allies

As shown in the previous section, overestimation of loss may improve the welfare of the allies in the second-best economy. We further investigate the normative role of misestimation to alleviate the free riding outcome of public good provision. If a country estimates its loss in the bad state than its true magnitude, it may purchase more security instruments to manage that risk. Moreover, suppose there exists a "World Government," which maximizes the social welfare of the alliance but cannot control the provision of public goods directly. If it chooses the level of bias in loss estimation such that the bias may induce countries to choose the socially optimal level of contribution to the public good, the countries voluntarily choose this optimal level in the Nash equilibrium. To summarize, we assume that the nature, or the "World Government," determines the bias in estimation and this section intends to investigate whether a social optimal bias exists. For the sake of simplicity, we omit self-insurance from the alliance model developed in section 2 and investigate the impacts of bias in estimation exclusively on self-protection and implied welfare.

3.1. Analytical Framework

Modifying our model to one public good model, we investigate how a country responds to a change in an assumed overestimation. Because we omit self-insurance, country A's maximization problem transforms as follows: A determines its provision of self-protection to maximize expected welfare subject to the following budget constraints:

$$\tilde{C}^{1A} = Y^A - m_1^A, \tag{32}$$

$$C^{0A} = Y^A - \bar{L}^A - \alpha^A - m_1^A. \tag{33}$$

Here, the magnitude of overestimations is exogenously assumed as in section 2.

Country A's maximization problem is summarized as follows:

$$\max_{m_1^A} \tilde{W}^A = pU(\tilde{C}^{1A}) + (1-p)U(\tilde{C}^{0A}),$$

subject to Eqs. (4), (5), (32), and (33).

The first order condition of the maximization problem for an interior solution is given as:

$$p'\left(\tilde{U}^{1A} - \tilde{U}^{0A}\right) - \left[p\tilde{U}_Y^{1A} + (1-p)\tilde{U}_Y^{0A}\right] = 0, \tag{34}$$

where $\tilde{U}^{iA} \equiv U(\tilde{C}^{iA}), \tilde{U}_Y^{iA} \equiv \partial U(\tilde{C}^{iA}) / \partial \tilde{C}^{iA}$ for $i = 0,1$. The first term on the left-hand side (LHS) of Eq.(34) shows country A's marginal benefit of self-protection, and the second term represents its marginal cost. A's contribution to self-protection benefits both country A and B. However, Eq. (34) does not include the marginal benefit of this self-protection for country B. In other words, A ignores the external benefit of country B. Thus, with no overestimation, A's contribution is less than the socially optimal level.

The second order condition is:

$$\frac{\partial^2 \tilde{W}^A}{\partial (m_1^A)^2} < 0. \tag{35}$$

We assume that the second order condition is satisfied. Solving Eq. (34), gives A's best response function, which depends on country B's contribution, A's income, and A's estimated loss:[7]

$m_1^A(m_1^B, Y^A, \tilde{L}^A)$.

Ihori and McGuire (2007) investigated the properties of such a best response function in a model without misestimations. The corresponding properties in this model are almost the same as in theirs, except that expected loss is not accurate, but biased. Taking the total differentiation of Eq. (34), gives the following:

$$\frac{\partial m_1^A}{\partial m_1^B} = \frac{-\dfrac{\partial^2 \tilde{W}^A}{\partial m_1^A \partial m_1^B}}{\dfrac{\partial^2 \tilde{W}^A}{\partial (m_1^A)^2}} = \frac{-1}{\dfrac{\partial^2 \tilde{W}^A}{\partial (m_1^A)^2}} \left\{ p''\left(\tilde{U}^{1A} - \tilde{U}^{0A}\right) + 2p'\left(\tilde{U}_Y^{0A} - \tilde{U}_Y^{1A}\right) + p\tilde{U}_{YY}^{0A} + (1-p)\tilde{U}_{YY}^{1A} \right\}, \tag{36}$$

$$\frac{\partial m_1^A}{\partial \tilde{L}^A} = \frac{-\dfrac{\partial^2 \tilde{W}^A}{\partial m_1^A \partial \tilde{L}^A}}{\dfrac{\partial^2 \tilde{W}^A}{\partial (m_1^A)^2}} = \frac{-1}{\dfrac{\partial^2 \tilde{W}^A}{\partial (m_1^A)^2}} \left\{ p'\tilde{U}_Y^{0A} + (1-p)\tilde{U}_Y^{1A} \right\}. \tag{37}$$

Then, we can deduce the following proposition.

Proposition 4

Suppose that country A purchases a positive amount of self-protection. Then, we have the following:

$$\frac{\partial m_1^A}{\partial m_1^B} < 0 \Leftrightarrow p''\left(\tilde{U}^{1A} - \tilde{U}^{0A}\right) - 2p'\tilde{U}_Y^{1A} + p\tilde{U}_{YY}^{1A} + \tilde{U}_Y^{0A}\left(2p' - (1-p)\tilde{R}^{0A}\right) < 0, \tag{38}$$

$$\frac{\partial m_1^A}{\partial \tilde{L}^A} > 0 \Leftrightarrow p'\tilde{U}_Y^{0A} + (1-p)\tilde{U}_{YY}^{0A} = \tilde{U}_Y^{0A}\left\{p' - (1-p)\tilde{R}^{0A}\right\} > 0, \tag{39}$$

where $\tilde{U}_{YY}^{iA} \equiv \partial^2 U(\tilde{C}^{iA}) / \partial(\tilde{C}^{iA})^2$ *and* $\tilde{R}^{0A} \equiv -\tilde{U}_{YY}^{0A} / \tilde{U}_Y^{0A}$.

Proposition 4 shows a comparative statics property of the best response function. Eq. (38)

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[7] Since the estimated loss is biased by misestimations, the best response does not maximize A's true expected welfare. In this sense, A's best response is not objectively "best" in its literal meaning, although it maximizes A's estimated (and biased) expected welfare.

implies that, if the absolute risk aversion in the bad state is sufficiently large, the contributions by countries to the self-protection public good are strategic substitutes in the sense that if country A (B) increases its contribution, B's (A's) national welfare maximizing contribution decreases. A sufficient condition for contributions to be strategic substitutes is that $2p' \leq (1-p)\tilde{R}^{0A}$. Even if this sufficient condition is not satisfied, self-protections may still be strategic substitutes if the first three terms on the LHS of Eq. (38) are sufficiently negative.

Equation (39) implies that, if the absolute risk aversion is sufficiently large, country A's self-protection increases with its estimated loss. An increase in the estimated loss has two effects on A's decision.

First, an increase in estimated loss raises the marginal benefit of self-protection. By reducing calculated utility in the bad state, it increases the welfare gap between the two states and raises the marginal benefit of self-protection.

Second, an increase in the estimated loss raises the marginal cost of self-protection. Because the country pays the cost of self-protection in both states of the world, its marginal cost is given as the probability-weighted sum of marginal utilities in both states. Because the additional loss reduces the income in the bad state, the government's budget constraint tightens and the marginal cost of self-protection evaluated in terms of utility rises.

The magnitude of the first effect depends on marginal utility, while the second effect depends on the second derivative of utility. Thus, comparing these two effects, if country A's absolute risk aversion is sufficiently small, the first effect dominates the second. Then, an increase in the estimated loss from the bad event will raise A's purchase of self-protection.

For example, assume the utility function is a constant relative risk aversion (CRRA) function. For such functions absolute risk aversion, $\tilde{R}^{0A}$, decreases with the consumption in the bad state. Since estimated consumption in the bad state decreases with estimated loss, the absolute risk aversion increases with the bias in loss estimation. When the bias is sufficiently small, $\tilde{R}^{0A}$ is so low that $\partial m_1^A / \partial \alpha^A$ is positive. But when the bias is sufficiently large, $\tilde{R}^{0A}$ becomes so high that $\partial m_1^A / \alpha^A$ is negative. Therefore, the curve representing m_1^A in (α^A, m_1^A)-space has an inverted U-shape.

Unlike in section 2, here we cannot not obtain a simple relationship between both countries' contributions, such as Eqs. (27), (28), and (29). Note that Propositions 1 depend on the fact that both countries contribute to the self-insurance public good. When a country purchases self-insurance, the ratio of marginal utility in the good state to that in the bad state is equal to the marginal insurance benefits. If country A contributes to self-protection, we rewrite the first order condition for self-protection as:

$$p'\frac{\left(\tilde{U}^{1A}-\tilde{U}^{0A}\right)}{\tilde{U}_Y^{0A}}=p\frac{\tilde{U}_Y^{1A}}{\tilde{U}_Y^{0A}}+(1-p)\,. \tag{40}$$

The LHS of Eq. (40) represents the marginal benefit of self-protection, and the RHS represents the marginal cost. If country A contributes to both self-protection and self-insurance, the ratio, $\tilde{U}_Y^{1A}/\tilde{U}_Y^{0A}$, in the first term on the RHS equals the marginal benefit of self-insurance. If both countries contribute to both self-protection and self-insurance, the RHS becomes identical for both countries. Then, consumption must be identical as well. However, if both countries do not contribute to self-insurance, the RHS is not necessarily identical for the two, so that in this case, we would not obtain a simple equation, such as Eqs. (27), (28), or (29).

3.2. Nash Equilibrium

We define the Nash equilibrium of our two-country game as the bundle of purchases of self-protection, (m_1^{A*}, m_1^{B*}), which satisfies the following equations:

$$m_1^{A*}=m_1^A(m_1^{B*},Y^A,\tilde{L}^A), \tag{41}$$

$$m_1^{B*}=m_1^B(m_1^{A*},Y^B,\tilde{L}^B). \tag{42}$$

Nash equilibrium level of self-protection and realized consumption of country A are given as:

$$M_1^*=m_1^{A*}+m_1^{B*}\,. \tag{43}$$

$$C^{1A*}=Y^A-m_1^{A*}, \tag{44}$$

$$C^{0A*}=Y^A-\bar{L}^A-m_1^{A*}. \tag{45}$$

The utility of households in country A is given as follows:

$$U^{1A*}=U(C^{1A*})\ \text{ and }\ U^{0A*}=U(C^{0A*})\,. \tag{46}$$

We refer to the “true” probability weighted realized utility as the true expected welfare:

$$W^{A*}=pU^{1A*}+(1-p)U^{0A*}. \tag{47}$$

However, because A maximizes its <u>estimated</u> expected welfare for any given level of B’s contribution to self-protection in Nash equilibrium, A does not maximize its <u>true</u> expected welfare. Nevertheless, the pessimistic bias in loss estimation provides additional incentives to contribute to self-protection and might cancel out free-riding incentives in both countries, just enough that they might lead to the socially optimal level of the true expected welfare.

Let us give an example wherein a pessimistic bias in loss estimation may improve the welfare of allies. Assume that the magnitude of the loss in the bad state is not devastating and that the absolute risk aversion decreases with income, as in the CRRA utility function. Then, the national income net of the loss in the bad state is large. Because the absolute risk aversion is decreasing, it is sufficiently small in the bad state such that an increase in bias of loss estimation may raise the provision of self-protection and improve the expected welfare.

To summarize, when the utility function shows a decreasing risk aversion, the overestimation of the loss (or positive bias in the loss estimation) may improve the welfare if the income net of the loss in the bad state is sufficiently large. On the other hand, if the net income is sufficiently small, the underestimation of the loss (or negative bias in the loss estimation) may improve the welfare.

3.3. Socially Optimal Estimation Bias

Although any bias in loss estimation leads to inefficient outcome in a single country model, such a bias may improve the welfare of allies in our alliance model because it mitigates the free-riding incentives. If biases in loss estimation induces allies to contribute more to the self-protection public good than they do in the absence of estimation bias, there may exist an optimal degree of bias such that it could induce allied countries to contribute the socially optimal amount of the self-protection public good. We refer to this optimal degree of bias as a “socially optimal degree of estimation bias” and discuss the sufficient condition of its existence.

First, we derive the socially optimal level of self-protection. Suppose that a social planner maximizes the utilitarian social welfare, $W^A + W^B$. Then, the first order conditions are given as the following:

$$p'\left(U^{1A}+U^{1B}-U^{0A}-U^{0B}\right)-\left[pU_Y^{1A}+(1-p)U_Y^{0A}\right]=0, \tag{48}$$

$$p'\left(U^{1A}+U^{1B}-U^{0A}-U^{0B}\right)-\left[pU_Y^{1B}+(1-p)U_Y^{0B}\right]=0. \tag{49}$$

The first term of Eq. (48) represents the marginal benefit of self-protection, which is the gain in the social welfare multiplied by the increase in the probability of a good state. The second term represents the marginal costs of self-protection. Because one unit of self-protection reduces country A’s consumptions in both states, the marginal cost is given as a probability-weighted sum of marginal utility of consumption. We assume that the second order condition is satisfied. Solving the first order conditions, we obtain the socially optimum levels of self-protection, m_1^{A**} and m_1^{B**}.

Second, we consider the existence of the socially optimal degree of bias. A vector of socially optimal estimation biases, $(\alpha^{A**}, \alpha^{B**})$, is defined such that, when countries A and B consider their loss in a bad state as $\bar{L}^A + \alpha^{A**}$ and $\bar{L}^B + \alpha^{B**}$, they voluntarily contribute socially optimal amounts, (m_1^{A**}, m_1^{B**}), to the self-protection public good. Formally, the vector $(\alpha^{A**}, \alpha^{B**})$ satisfies the following system of equations:

$$\begin{aligned} & p'^{**}\left[U(Y^A - m_1^{A**}) - U(Y^A - m_1^{A**} - \bar{L}^A - \alpha^{A**})\right] \\ & -\left[p^{**}U_Y(Y^A - m_1^{A**}) + (1-p^{**})U_Y(Y^A - m_1^{A**} - \bar{L}^A - \alpha^{A**})\right] = 0, \end{aligned} \tag{50}$$

$$\begin{aligned} & p'^{**}\left[U(Y^B - m_1^{B**}) - U(Y^B - m_1^{B**} - \bar{L}^B - \alpha^{B**})\right] \\ & -\left[p^{**}U_Y(Y^B - m_1^{B**}) + (1-p^{**})U_Y(Y^B - m_1^{B**} - \bar{L}^B - \alpha^{B**})\right] = 0, \end{aligned} \tag{51}$$

where $p^{**} \equiv p(m_1^{A**} + m_1^{B**}), p'^{**} \equiv p'(m_1^{A**} + m_1^{B**})$. Equation (50) is the first order condition for A to maximize its expected welfare. Countries A and B purchase the socially optimum amounts of self-protection, m_1^{A**} and m_1^{B**}. Without overestimations, or $\alpha^{A**} = 0$, the LHS of Eq. (50) is negative. If an optimal value of estimation bias, α^{A**}, exists such that it solves Eq. (50), then country A voluntarily contributes m_1^{A**} in response to the socially optimal degree of bias, α^{A**}.

We cannot straightforwardly derive the sufficient condition for the existence of the solution of Eqs. (50) and (51), $(\alpha^{A**}, \alpha^{B**})$, to exist. Here, we assume that both countries are endowed with the identical income, loses the identical amount in case of the bad state, biased by the same level in their estimation of the loss, and limit ourselves to consider a symmetric Nash equilibrium wherein both countries contribute the same amount to the public good. Then, we have $Y^A = Y^B = Y, \bar{L}^A = \bar{L}^B = \bar{L}, m_1^{A**} = m_1^{B**} = m_1^{**}$, and $\alpha^{A**} = \alpha^{B**} = \alpha^{**}$. The symmetric optimal self-protection and the socially optimal estimation bias, (m_1^{**}, α^{**}), should satisfy the following system of equations:

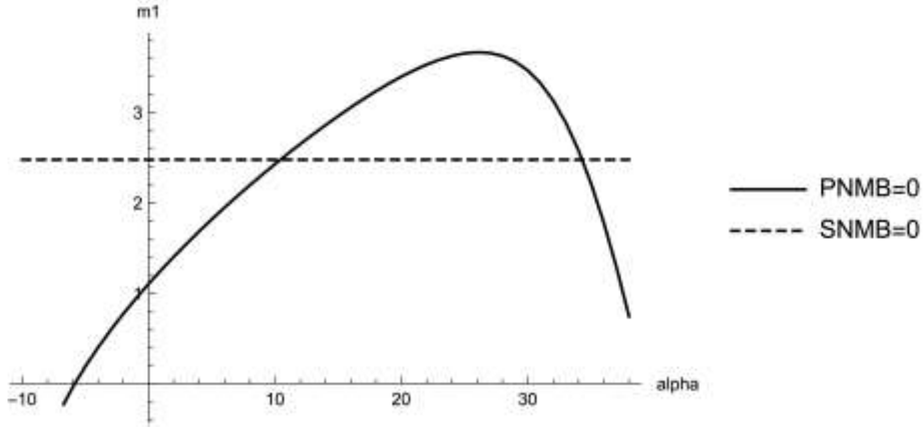
$$\begin{aligned} SNMB(m_1^{**}) \equiv\ & 2p'(2m_1^{**})\left[U(Y - m_1^{**}) - U(Y - m_1^{**} - \bar{L})\right] \\ & -\left[p(2m_1^{**})U_Y(Y - m_1^{**}) + (1 - p(2m_1^{**}))U_Y(Y - m_1^{**} - \bar{L})\right] = 0, \end{aligned} \tag{52}$$

$$PNMB(m_1^{**},\alpha^{**}) \equiv p'(2m_1^{**})\Big[U(Y-m_1^{**})-U(Y-m_1^{**}-\bar{L}-\alpha^{**})\Big] \\ -\Big[p(2m_1^{**})U_Y(Y-m_1^{**})+(1-p(2m_1^{**}))U_Y(Y-m_1^{**}-\bar{L}-\alpha^{**})\Big]=0, \tag{53}$$

where $SNMB(m_1^{**})$ represents the Social Net Marginal Benefit (SNMB) of self-protection, which is the marginal increase in the utilitarian social welfare from a marginal increase in self-protection, and $PNMB(m_1^{**},\alpha^{**})$ the Private Net Marginal Benefit (PNMB) of self-protection, which is the marginal increase of the welfare of each country from a marginal increase in self-protection.[8] Both $SNMB(m_1^{**})$ and $PNMB(m_1^{**},\alpha^{**})$ are evaluated when each country provides m_1^{**} units of self-protection and overestimates its loss in the bad state by α^{**} units. If m_1^{**} maximizes the utilitarian social welfare, the SNMB must be zero. We note that Eq. (52) does not include any bias in loss estimation, which implies that the optimal self-protection, m_1^{**}, is independent of the overestimation. If a country maximizes its estimated expected welfare, the PNMB must be zero. When m_1^{**} and α^{**} satisfy Eq. (53), each country maximizes its biased expected welfare by choosing m_1^{**} as its amount of self-protection.

[8] Both countries, here, have the same utility function, face the same loss in a bad state and are endowed with the same income. Thus, the PNMB of A is identical to that of B.

Figure 1: Multiple optimal overestimations



Note: PNMB: private net marginal benefit; SNBM: social net marginal benefit
Source: Authors.

Figure 1 pictures how the value of α^{**} is determined. The horizontal axis of this figure represents the bias in loss estimation, while its vertical axis represents the purchase of self-protection. This figure is drawn based on the functional forms and parameters in the numerical examples reported in appendix C. Utility is a CRRA function, while the probability of the good state follows the contest success function developed by Tullock (1967). The dashed line represents the solution of Eq. (52). Since the socially optimal self-protection is not influenced by any bias in estimation, it is horizontal. The solid curve gives the locus of points satisfying Eq. (53). As shown in Figure 1, when there is no bias in loss estimation ($\alpha = 0$), each country voluntarily purchases less self-protection than the socially optimal level.

In Figure 1, the voluntary provision of self-protection increases with the estimation bias when that bias is small. Then, the solid curve intersects the dashed line, where the bias induces countries to purchase the socially optimal level of self-protection. Further bias leads to overprovision of self-protection. Moreover, when the solid curve passes its peak, additional bias reduces the voluntary purchase of self-protection. Then, the solid curve intersects the dashed line again. In Figure 1, there are two intersections of the solid curve and the dashed line. The abscissas of the intersections represent the levels of estimation bias that make countries voluntarily purchase the socially optimal amount of self-protection. Thus, we can consider these levels as socially optimal estimation biases.

Generally, multiple values of socially optimal biases do not necessarily exist. Countries might not sufficiently increase their voluntary purchase in response to the overestimation of loss. Then, the solid curve may not intersect the dashed curve at all. In this case, no socially optimal bias exists because no estimation bias could induce countries to provide the socially optimal amount of self-protection.

4: Endogenous Misestimation and the Normative Role of an NGO in Identical Allies

Although overestimations of loss may improve the welfare of allies in our alliance model, the allies do not have incentives to overestimate. In reality, however, it is also unlikely that the governments precisely estimate the loss in a bad event because many agents, such as lobbyists, activists, politicians, bureaucrats, and journalists, may influence the governments' estimations. In this section, we focus on the influences of international NGOs because they have increased their presence in the formation of foreign policies and international agreements in the last decades.

For example, let us consider self-protection as diplomatic efforts including foreign aids. Each allied country may offer foreign aids to developing countries not only to make them friendly to the alliance but to prevent a humanitarian crisis in these countries, which may lead to a surge of refugees to the allied countries. Because of the free-riding incentive, such aids would be less than the socially optimal level. International NGOs might extract additional aids or supports to developing world from these governments by emphasizing the threat of humanitarian crisis. We may explain that these NGOs are trying to increase the governments' self-protection through emphasizing the damage in a bad state.

Another example is a pro-environmental group. Although evidences of climate change are increasing, several countries have been reluctant to conduct strict environmental policy, such as introducing heavy rate of carbon taxes and restricting construction of coal power plants. We may explain that these countries free-ride in preventing climate change. Pro-environmental groups are trying to change the policy of these countries by emphasizing the impact of climate change. We may interpret these NGOs' behavior as lobbying to bias governments' estimation of their loss in case of severe climate change. We discuss the impacts of such behavior of NGOs on the welfare of allied countries.

We first conduct a positive analysis of the behavior of the NGO. As before, we consider two identical allied countries. We incorporate a hypothetical NGO into our model. In the real world, NGOs have various objectives. However, we concentrate on NGOs that benefit from the provision of self-protection. For example, additional foreign aids facilitate activities of humanitarian NGOs. Then, we may argue that these NGOs benefit from foreign aids. We therefore assume that the NGO benefits from the provision of the self-protection public good and that it maximizes its net benefit, which is defined as its benefit minus its lobbying cost. We also assume that the NGO may influence the loss estimation by all allied governments through the NGO's lobbying efforts. In addition, we assume that the NGO is located outside the alliance, which implies that the governments of the allied countries ignore the net benefit of NGO. Even if the NGO is geographically located in the alliance, our analysis is applicable as far as the governments ignores its payoff in their decision making. This is the simplest theoretical framework to formulate the NGO's behavior to derive meaningful analytical results.

The model has three periods. The time structure is summarized as follows:

Period 0: An international NGO determines its lobbying efforts in the two countries to maximize its net benefit. The lobbying biases the loss estimation by the allied countries. The estimated loss of each country increases with the NGO's lobbying effort in the country.

Period 1: Each country independently and simultaneously determines its purchase of self-protection to maximize its estimated expected welfare based on the estimated loss.

Period 2: The state of the world is stochastically determined based on the self-protection provided in Period 1. The consumptions of households in both countries are realized.

We solve this game using backward inductions. In Period 2, the state of the world is chosen, where the probability of good or bad states is determined by the provision of self-protection, as per Eq. (43). The consumptions of country A in good and bad states, C^{1A*} and C^{0A*}, are given by Eqs. (44) and (45).

In Period 1, each country non-cooperatively maximizes its expected welfare. The governments of A and B do not know their true loss in the bad state. Country A estimates its loss in the bad state as $\tilde{L}^A$, where $\tilde{L}^A = \bar{L}^A + \alpha^A$. B estimates its loss as $\tilde{L}^B$, where $\tilde{L}^B = \bar{L}^B + \alpha^B$. Country A estimates its budget constraints in Period 2 as Eqs. (32) and (33). It maximizes its expected welfare subject to its estimated budget constraints. The first order condition is given by Eq. (34). The Nash equilibrium of the game in Period 1 is then given by Eqs. (41) and (42). Because the equilibrium allocation of self-protection is given as a function of estimated losses, we denote the Nash equilibrium in Period 1 as $m_1^{A*}(\tilde{L}^A, \tilde{L}^B)$ and $m_1^{B*}(\tilde{L}^B, \tilde{L}^A)$.

In Period 0, the international NGO determines its lobbying efforts to maximize its net benefit. For the sake of simplicity, we assume that the benefit of the NGO is proportional to the amount of self-protection public good and that its lobbying cost is proportional to the level of bias. Thus, its net benefit is given as follows:

$$\Pi = \beta(m^{A*} + m^{B*}) - c(\alpha^A + \alpha^B), \tag{54}$$

where β is the coefficient of the benefit of self-protection and c is the constant unit cost of lobbying. The unit cost of lobbying is the cost required for increasing the estimated loss by a government by one unit of loss. We also assume that the cost of lobbying does not affect the budget constraints of the allied countries. Although this is a strong but simplifying assumption, if the NGO tries to influence the public opinion in the allied countries through the Internet from the outside of the alliance, the cost of lobbying may not affect the budget constraints of these countries.

The interior first order condition for the NGO is given as follows:

$$\beta\left(\frac{\partial m_1^{A*}}{\partial \tilde{L}^A}+\frac{\partial m_1^{B*}}{\partial \tilde{L}^A}\right)=c, \tag{55}$$

$$\beta\left(\frac{\partial m_1^{A*}}{\partial \tilde{L}^B}+\frac{\partial m_1^{B*}}{\partial \tilde{L}^B}\right)=c\ . \tag{56}$$

The partial derivatives of $m_1^{A*}(.)$ and $m_1^{B*}(.)$ are derived by taking the total differentiation of Eqs. (41) and (42). Details of the derivation are given in appendix B of this paper. The subgame perfect Nash equilibrium of this model is given as the solution of the system of Eqs. (41), (42), (55), and (56).

Equations (55) and (56) implies that the equilibrium levels of estimation biases depend on the unit cost of lobbying. This cost may decrease with a progress in information technologies and an opening of the international policy arena to NGOs. We consider whether there exists the optimal level of the unit cost of lobbying. For the sake of simplicity, we assume that both countries have identical preferences, that their incomes are identical and that their losses are also identical. We define the optimal unit cost of lobbying, c^{**} as a cost such that the NGO voluntarily chooses the socially optimal degree of estimation bias. It must satisfy the following equation:

$$\left.\frac{\partial m_1^{A*}}{\partial \tilde{L}^A}\right|_{m_1^A=m_1^B=m_1^{**},\alpha^A=\alpha^B=\alpha^{**}}+\left.\frac{\partial m_1^{A*}}{\partial \tilde{L}^B}\right|_{m_1^A=m_1^B=m_1^{**},\alpha^A=\alpha^B=\alpha^{**}}=\frac{c^{**}}{\beta}, \tag{57}$$

where m_1^{**} is the socially optimal level of self-protection contribution and α^{**} is the socially optimal degree of estimation bias that induces countries to contribute the socially optimal amount to the self-protection public good.

Finally, we conduct several numerical simulations to explore potential roles and limitations of NGOs. Here, we report the main result of this simulation. The details of results are reported in appendix C. We show that there exists a degree of estimation bias that induces countries voluntarily contribute the socially optimal amount of self-protection public good. In this case, the welfare of countries in the Nash equilibrium are equal to those in the social optimum. We also show that in a certain range of loss, there may exist two values of socially optimal estimation bias as shown in Figure 1. Even if there are two values, the lower value is more relevant for NGOs than the higher value, simply because lobbying politicians and advertising to the public are costly, and NGOs are concerned with net benefits. Then, the lower socially optimal bias increases with the true loss. We therefore may conclude that as the threat of a bad state becomes more severe, the socially optimal bias becomes larger. If the loss in a bad state is sufficiently large, any degree of esimation bias may not derive the socially optimal behavior of the countries.

Our result explores the beneficial role of NGOs. If NGOs may influence governments' estimations of loss and induce them to contribute more to public goods, their intervention in policymaking is beneficial, that is, their efforts may improve the welfare of all allies. It may even be possible for the NGO to induce countries to contribute the socially optimal amount of public good. However, there is a limitation in the role of NGOs. When the true losses are sufficiently large, the difference between the social optimal and the Nash equilibrium becomes so wide that any estimation bias cannot induce countries to contribute the socially optimal amount of the public good. Thus, even if NGOs spread an extremely pessimistic estimation of the bad state, the socially optimal provision of self-protection cannot be achieved in this case.

We also conduct simulations to investigate the relationship between the optimal unit cost of lobbying and the loss in a bad state. The detailed results of simulations are also reported in appendix C. The main results are summarized as follows: (1) There is a range of loss such that the optimal unit cost of lobbying decreases with this loss. (2) For losses larger than the upper bound of the range, there exists no unit cost of lobbying that induces NGOs to choose the socially optimal bias. (3) For losses smaller than the lower bound of the range, the socially optimal amount of self-protection becomes zero, which coincides with the Nash equilibrium level. Then, any lobbying or estimation bias is not required.

Because the unit cost of lobbying may decrease with the openness of the international policy arena, our result implies that we should be more open to NGOs when the severity of a threat for allied countries increases, at least if the severity of the threat is moderate. However, if the threat is extremely severe, the normative role of NGOs is limited, that is, any estimation bias cannot attain the social optimum. Nevertheless, the NGOs could alleviate the free-riding behavior of allied countries to some extent even in this case.

5: Conclusion

In this paper, we investigated the impacts of misestimations of the severity of a threat on collective risk management. Here, we formulated the misestimation as a bias in the estimation of a loss in the bad event. We addressed the following three questions: (1) How would a bias in loss estimation affect burden-sharing among allies? (2) Whether does there exist a degree of bias that induces governments to voluntarily provide a socially optimal amount of a self-protection public good? (3) Could an international NGO voluntarily choose the socially optimal level of lobbying to influence governments' estimation of losses, and induce these governments to provide the socially optimal level of self-protection?

In section 2, we constructed a two-country model wherein two countries voluntarily provide self-protection and self-insurance public goods. Then, we investigated the impacts of an overestimation (or underestimation) of the loss in a bad state on the consumptions and contributions of the countries. In an interior Nash equilibrium, compared to the other ally, a country that is more biased in estimation consumes more in the bad state. We also showed that the difference in total security expenditure between allies is not influenced by misestimations. However, the composition of security expenditure is influenced by the estimation bias. To elaborate, the difference in self-protection (the difference in self-insurance) between two allied countries becomes smaller (larger) than it is in the absence of any estimation bias. This shrinking of the difference in self-protection expenditure is canceled out by the widening of the difference in self-insurance expenditure. Using numerical simulations, we showed that overestimations of losses may improve the welfare of both countries in our two-country model. The overly estimated losses increase the incentive to contribute, which may cancel out the free-riding incentives in voluntary provision of public good.

In section 3, we conducted a normative analysis of misestimation. We simplified our model such that we only considered a voluntary provision of a self-protection public good. We investigated how much bias in loss estimations maximize the utilitarian social welfare of allies. In a one-country model, estimation bias would simply hinder the government in achieving the optimal supply of self-protection. However, in the alliance model, the bias in loss estimation may stimulate the provision of the public good and mitigate the free-riding problem in the Nash equilibrium.

In section 4, we introduced an international NGO that can affect the loss estimations of governments. We assumed that one international NGO locates outside the alliance and that it lobbies for more pessimistic estimation of the loss in a bad state before the government's decision-making. We also assumed that the NGO benefits from the provision of self-protection public good and pays the cost of lobbying and that it maximizes its net benefit through lobbying. Our numerical simulations explored the possibility of the socially optimal estimation bias and its relationship with the true loss in the bad state. We showed that the optimal bias may increase with the loss unless the loss is not extremely large and that the optimal marginal lobbying cost should decrease with the loss of the bad event.

Our analysis sheds new light on the normative role of the NGOs. In international risk management, we cannot presuppose the “World Government” that achieves the optimal provision of the public goods by collecting some tax from the players and paying matching grants for the provision of public goods. However, intentionally or unintentionally, international NGOs may pessimistically bias the government’s estimation of loss in the bad state. This then increases the equilibrium level of the self-protection public good and improves the welfare of the allies. To summarize, our result explores the beneficial role of NGOs, that is, as long as the difference between the social optimum and the Nash equilibrium exists, misestimation caused by the lobbying efforts of NGOs could improve the social welfare.

A: Proofs of Propositions 1 and 2

A.1 Proof of Proposition 1

In this proof, we solve the first order conditions of the optimal self-protection and self-insurance to define the optimal consumptions in the good and bad states as a function of provisions of public goods. Because both countries consume the identical amounts of public goods, their consumptions also become identical.

First, we rewrite the first order conditions of self-insurance and self-protection contributions, Eqs. (14) and (15), as follows:

$$p'\left(U(\tilde{C}^{1A})-U(\tilde{C}^{0A})\right)-\left\{pU_Y(\tilde{C}^{1A})+(1-p)\left(1-\mathrm{L}'\frac{\partial s}{\partial M_1}\right)U_Y(\tilde{C}^{0A})\right\}=0\,, \tag{58}$$

$$\mathrm{L}'U_Y(\tilde{C}^{0A})-U_Y(\tilde{C}^{1A})=0\,. \tag{59}$$

In the conventional model of the voluntary provision of public goods, players choose their private good consumption and contribution to the public good. In our model, governments do not directly choose their private good consumptions or public good provisions. Thus, it is not straightforward that we can solve the system of Eqs. (58) and (59) to obtain $\tilde{C}^{0A}$ and $\tilde{C}^{1A}$ as functions of M_1 and M_2.

The procedure of solving Eqs. (58) and (59) is as follows. First, we solve Eq. (59) to obtain the optimal consumption in the good state as a function of the consumption in the bad state and provision of public goods. Second, we substitute the optimal consumption in the good state for the first order condition of the optimal self-protection to obtain the optimal consumption in the bad state as a function of the provision of public goods. Finally, we show that because public goods for country A are identical to those for country B in the Nash equilibrium, the consumption of country A is also identical to that of country B.

The first process is to derive the optimal consumption in the good state as a function of consumption in the bad state and the provision of public goods. We rewrite Eq. (15) as:

$$L'\left(\frac{pM_2}{1-p}\right)U_Y(\tilde{C}^{0A})-U_Y\left(\tilde{C}^{1A}\right)=0\,. \tag{60}$$

We interpret Eq. (60) as an implicit function of $\tilde{C}^{1A}$ for any given $\tilde{C}^{0A}$, M_1 and M_2. Because $U_{YY}<0$, we derive the optimal consumption function in the good state as follows:

$$\tilde{C}^{1A}=e(\tilde{C}^{0A},M_1,M_2)\,. \tag{61}$$

Because the utility function of country B is identical to that of A, we also obtain:

$$\tilde{C}^{1B}=e(\tilde{C}^{0B},M_1,M_2)\,. \tag{62}$$

Function $e(.)$ has the following properties. From Eq. (60), we obtain:

$$\frac{U_Y^A(\tilde{C}^{1A})}{U_Y^A(\tilde{C}^{0A})}=L'\,. \tag{63}$$

Because we assume $\mathrm{L}'\leq 1$ and $U_{YY}<0$, we obtain:

$$e(\tilde{C}^{0A},M_1,M_2)=\tilde{C}^{1A}\geq\tilde{C}^{0A}. \tag{64}$$

By definition, it follows that:

$$U_Y\left(e(\tilde{C},M_1,M_2)\right)=L'\left(\frac{pM_2}{1-p}\right)U_Y(\tilde{C})\quad\text{for any }\ \tilde{C}\,. \tag{65}$$

We then obtain the partial derivative of function $e(.)$ as:[9]

$$\frac{\partial e(\tilde{C},M_1,M_2)}{\partial\tilde{C}}=\frac{\mathrm{L}'U_{YY}(\tilde{C})}{U_{YY}(e(\tilde{C},M_1,M_2))}>0\,. \tag{66}$$

The second process is to obtain the consumption in the bad state as a function of the provisions of the public goods. We rewrite Eq. (58) as:

$$\frac{U(\tilde{C}^{1A})-U(\tilde{C}^{0A})}{U_Y(\tilde{C}^{0A})}=\frac{1}{p'}\left\{p\,\mathrm{L}'+(1-p)\left(1-\mathrm{L}'\frac{\partial s}{\partial M_1}\right)\right\}. \tag{67}$$

Substituting Eq. (61) into Eq. (67), we obtain:

$$\frac{U(e(\tilde{C}^{0A},M_1,M_2))-U(\tilde{C}^{0A})}{U_Y(\tilde{C}^{0A})}=\frac{1}{p'}\left\{p\,\mathrm{L}'+(1-p)\left(1-\mathrm{L}'\frac{\partial s}{\partial M_1}\right)\right\}. \tag{68}$$

The left-hand side (LHS) of Eq. (68) represents the marginal benefit of the increasing probability of the good state and the right-hand side (RHS) is its marginal cost. We take the partial differentiation

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[9] Using Eq. (60), we obtain:

$$\frac{\partial e(C,M_1,M_2)}{\partial C}<1\ \text{ if and only if }\ -\frac{U_{YY}(C)}{U_Y(C)}<-\frac{U_{YY}(e(C,M_1,M_2))}{U_Y(e(C,M_1,M_2))}\,.$$

Thus, the slope of the curve of function $e(.)$ in $\tilde{C}^{0A}-\tilde{C}^{1A}$ space is steeper (more gradual) than the unity if the absolute risk aversion increases (decreases).

of the LHS, utilize Eq. (22), and obtain the following:[10]

$$\frac{\partial}{\partial \tilde{C}^{0A}} \frac{U(e(\tilde{C}^{0A}, M_1, M_2)) - U(\tilde{C}^{0A})}{U_Y(\tilde{C}_Y^{0A})} = \frac{-(U(e) - \tilde{U}^{0A})\tilde{U}_{YY}^{0A}}{(\tilde{U}_Y^{0A})^2} + \frac{1}{\tilde{U}_Y^{0A}} \left\{ U_Y(e) \frac{\partial e}{\partial \tilde{C}^{0A}} - \tilde{U}_Y^{0A} \right\}. \quad (69)$$

Thus, the first term on the RHS of Eq. (69) is positive. The sign of the brackets of the second term on the RHS is given as follows:

$$U_Y(e) \frac{\partial e}{\partial \tilde{C}^{0A}} - \tilde{U}_Y^{0A} = \tilde{U}_Y^{1A} \frac{\mathrm{L}' \tilde{U}_{YY}^{0A}}{\tilde{U}_{YY}^{1A}} - \tilde{U}_Y^{0A} = \tilde{U}_Y^{0A} \left(\frac{\mathrm{L}' \tilde{R}^{0A}}{\tilde{R}^{1A}} - 1 \right) > 0. \quad (70)$$

The last inequality of Eq. (70) is derived from our assumption in Eq. (22).

The final process is to apply a similar reasoning to the first order condition of country B to obtain:

$$\frac{U(e(\tilde{C}^{0B}, M_1, M_2)) - \tilde{U}^{0B}}{\tilde{U}_Y^{0B}} = \frac{1}{p'} \left\{ p\,\mathrm{L}' + (1-p)\left(1 - \mathrm{L}' \frac{\partial s}{\partial M_1}\right) \right\}, \quad (71)$$

where the LHS of Eq. (71) increases with C^{0B}. Comparing Eq. (68) with Eq. (71), we obtain:

$$\frac{U(e(\tilde{C}^{0A}, M_1, M_2)) - U(\tilde{C}^{0A})}{U_Y(\tilde{C}^{0A})} = \frac{U(e(\tilde{C}^{0B}, M_1, M_2)) - U(\tilde{C}^{0B})}{U_Y(\tilde{C}^{0B})}. \quad (72)$$

Because the LHS of Eq. (72) increases with $\tilde{C}^{0A}$ and the RHS increases with $\tilde{C}^{0B}$, we obtain:

$$\tilde{C}^{0A} = \tilde{C}^{0B}. \quad (73)$$

Substituting Eq. (73) into Eqs. (61) and (62), we have $\tilde{C}^{1A} = \tilde{C}^{1B}$.

A.2 Proof of Proposition 2

We remember that the Nash equilibrium allocation $(m_1^{A*}, m_2^{A*}, m_1^{B*}, m_2^{B*})$ satisfies the budget constraints given by Eqs. (7) and (8). Country B's budget constraints are given by replacing superscript A with B in Eqs. (7) and (8). The Nash equilibrium levels of contributions made by country B (m_1^{B*} and m_2^{B*}) satisfy B's budget constraints. Substituting the budget constraints into Eq. (23), we obtain

$$Y^A - m_1^{A*} - m_2^{A*} = Y^B - m_1^{B*} - m_2^{B*}. \quad (74)$$

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[10] From Eq. (64), the first term on the RHS of Eq. (69) is non-negative. Substituting Eq. (66) into the second term and using our assumption, Eq. (22), we obtain:

$$U_Y(e) \frac{\partial e}{\partial \tilde{C}^{0A}} - \tilde{U}_Y^{0A} = U_Y(e) \frac{\mathrm{L}' \tilde{U}_{YY}^{0A}}{\tilde{U}_{YY}(e)} - \tilde{U}_Y^{0A} > 0.$$

$$Y^A - \bar{L}^A - \alpha^A - m_1^{A*} + \mathrm{L}(s) = Y^B - \bar{L}^B - \alpha^B - m_1^{B*} + \mathrm{L}(s), \tag{75}$$

Substituting Eq. (12) into Eq. (74), we obtain Eq. (24). Substituting Eq. (13) into Eq.(75), we obtain

$$C^{0A*} - \alpha^A = C^{0B*} - \alpha^B. \tag{76}$$

Rearranging Eq. (76), we obtain Eq. (25).

To show Eq. (26), we first assume that $\alpha^A > \alpha^B$. Then, Eq. (25) implies that $C^{0A*} > C^{0B*}$. Using Eq. (24), we obtain

$$W^{A*} = p(M_1^*)U(C^{1A*}) + (1 - p(M_1^*))U(C^{0A*}) > p(M_1^*)U(C^{1B*}) + (1 - p(M_1^*))U(C^{0B*}) = W^{B*}.$$

Thus, the sufficient condition of Eq. (26) is established. Next, we assume that $W^{A*} > W^{B*}$. Then, we obtain

$$W^{A*} = p(M_1^*)U(C^{1A*}) + (1 - p(M_1^*))U(C^{0A*}) > p(M_1^*)U(C^{1B*}) + (1 - p(M_1^*))U(C^{0B*}) = W^{B*}. \tag{77}$$

Using Eq. (24) and our assumption that $p(M_1) \in (0,1)$, Eq. (77) reduces to

$$U(C^{0A*}) > U(C^{0B*}). \tag{78}$$

Since $U' > 0$ and $C^{0A*} = \tilde{C}^{0A*} + \alpha^A$, we have

$$\tilde{C}^{0A*} + \alpha^A > \tilde{C}^{0B*} + \alpha^B. \tag{79}$$

Substituting Eq. (23) into Eq. (79), we obtain the necessary condition of Eq. (26).

B: Comparative Statics of Nash Equilibrium in Period 1

In this appendix, we conduct the comparative statics of the Nash equilibrium in Period 1 in the model constructed in section 4.1. We take the total differentiation of Eqs. (41) and (42), and solve the resulting simultaneous equations. Then, we obtain the following:

$$dm_1^{A*} = \frac{1}{D}\left(\frac{\partial m_1^A}{\partial \tilde{L}^A} d\tilde{L}^A + \frac{\partial m_1^A}{\partial m_1^B} \frac{\partial m_1^B}{\partial \tilde{L}^B} d\tilde{L}^B \right), \tag{80}$$

$$dm_1^{B*} = \frac{1}{D}\left(\frac{\partial m_1^B}{\partial \tilde{L}^B} d\tilde{L}^B + \frac{\partial m_1^B}{\partial m_1^A} \frac{\partial m_1^A}{\partial \tilde{L}^A} d\tilde{L}^A \right), \tag{81}$$

where $D \equiv 1 - \frac{\partial m_1^A}{\partial m_1^B} \frac{\partial m_1^B}{\partial m_1^A}$. We substitute Eqs. (36) and (37) in Eq. (80), and assume $d\tilde{L}^B = 0$ to obtain the following:

$$\frac{\partial m_1^{A*}}{\partial \tilde{L}^A} = \frac{-1}{\Delta} \frac{\partial^2 \tilde{W}^B}{\partial (m_1^B)^2} \frac{\partial^2 \tilde{W}^A}{\partial m_1^A \partial \tilde{L}^A}, \tag{82}$$

where $\Delta \equiv \frac{\partial^2 \tilde{W}^A}{\partial (m_1^A)^2} \frac{\partial^2 \tilde{W}^B}{\partial (m_1^B)^2} - \frac{\partial^2 \tilde{W}^A}{\partial m_1^A \partial m_1^B} \frac{\partial^2 \tilde{W}^B}{\partial m_1^B \partial m_1^A}$. Assuming $d\tilde{L}^B = 0$ in Eq. (81), we obtain the following:

$$\frac{\partial m_1^{B*}}{\partial \tilde{L}^A} = \frac{1}{\Delta} \frac{\partial^2 \tilde{W}^B}{\partial m_1^B \partial m_1^A} \frac{\partial^2 \tilde{W}^A}{\partial m_1^A \partial \tilde{L}^A} . \tag{83}$$

C: Numerical Examples of Endogenous Misestimation

In this appendix, we conduct several numerical simulations to consider the following two questions: (1) Whether is there a socially optimal degree of bias in loss estimation that may effectively induce allied countries to contribute the socially optimal amount of self-protection? (2) How low is the unit cost of lobbying enough low for the NGO to cause the socially optimal degree of bias in governments' estimations?

C.1 Specification for Simulations

To conduct numerical analysis, we specify the forms of the functions in our model. We follow Ihori et al. (2014) in the specification. We specify the utility function, $U(.)$, as a CRRA function:

$$U(C) = \frac{C^{1-\theta}}{1-\theta} , \tag{84}$$

where θ is the parameter representing the relative risk aversion of the country. We also specify the probability function following Tullock's contest success function:

$$p(M_1) = \frac{M_1 + p_e}{M_1 + (p_e / p_0)}, \tag{85}$$

where $p_0 (> 0)$ is the baseline probability of the good state, which is the probability of the good state when no country provides any self-protection public good, and $p_e (> 0)$ is a parameter representing the strength of the opponent.

For the sake of simplicity, we assume that both countries are identical in the sense that they have identical preferenses, income and contingent loss in a bad state.

C.2 Socially Optimal Estimation Bias

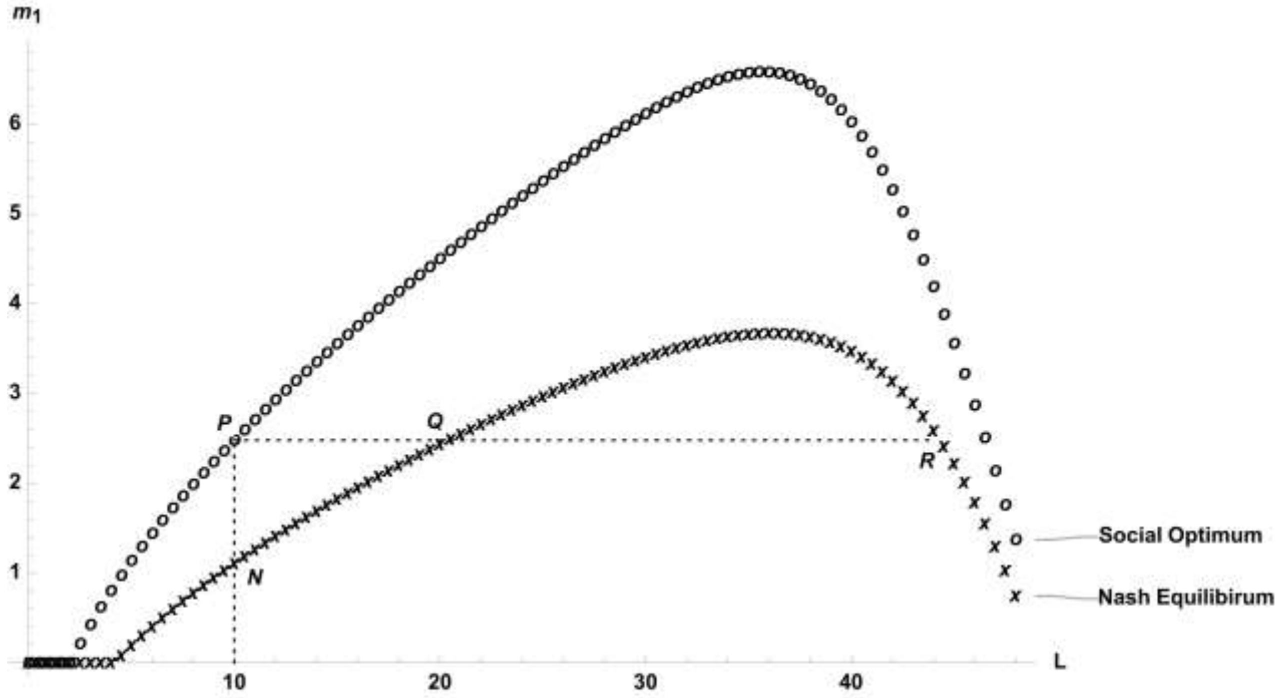
Table 3 Numerical simulations of the socially optimal estimation bias

Scenario	1 Social optimum	2 Nash equilibrium without estimation bias	3a Nash equilibrium with socially optimal estimation bias	3b Nash equilibrium with socially optimal estimation bias
Y^A	50	50	50	50
Y^B	50	50	50	50
$\bar{L}^A$	10	10	10	10
$\bar{L}^B$	10	10	10	10
α^A	n/a	0	10.429	34.301
α^B	n/a	0	10.429	34.301
p_0	0.25	0.25	0.25	0.25
p_e	1	1	1	1
θ	0.9	0.9	0.9	0.9
m_1^A	2.480	1.105	2.480	2.480
m_1^B	2.480	1.105	2.480	2.480
M_1	4.960	2.210	4.960	4.960
$\tilde{W}^A$	n/a	14.540	14.354	13.162
$\tilde{W}^B$	n/a	14.540	14.354	13.162
W^A	14.559	14.540	14.559	14.559
W^B	14.559	14.540	14.559	14.559

Source: Authors.

Table 3 presents three numerical examples of our two-period game in section 3 wherein the bias in loss estimation is given as a paramter. In this model, the loss in the bad state is 20% of the income. Column 1 of this table reports the solution of the utilitarian social welfare maximization problem. Under the settings of the parameters in this table, the optimal purchase of self-protection is 2.480, which is 4.9% of the income. Column 2 represents the Nash equilibrium allocation of this model wherein no bias in estimation exists. In this case, each country voluntarily purchases 1.105 units of self-protection, which is less than half of the optimal level. Columns 3a and 3b report the allocations in the Nash equilibria in the presence of the socially optimal estimation bias. As shown in section 3.3, there are two values of socially optimal estimation bias that induce countries to provide the socially optimal level of the self-protection public good, which are 10.429 in column 3a and 34.301 in column 3b, respectively. In both cases, each country purchases 2.480 units of self-protection, which is socially optimal. The levels of true expected welfare in columns 3a and 3b are equal to those in the socially optimum scenario.

Figure 2 Self-protection contributions in the social optimum and the Nash equilibrium



Source: Authors.

To investigate how an estimation bias induces a country to contribute the socially optimal self-protection, we examine the relationship between loss in the bad state and the governments' choice of self-protection. Figure 2 illustrates how the socially optimum level and the Nash equilibrium level of self-protection contributions vary with loss in the bad state, where there is no bias in estimation, or $\alpha^A = \alpha^B = 0$. The parameters are set as in Table3, except for loss in the bad state and the estimation bias. The two identical countries face the same amount of loss in the bad state, $\bar{L}^A = \bar{L}^B = L$, and L increases from 0 to 48. The horizontal axis represents the value of loss, and the vertical axis represents the amount of self-protection provided by each country. Each "o"-marker represents the socially optimum level of the self-protection contribution, which maximizes the utilitarian social welfare of the two countries. Each "x"-marker represents the level of contribution in the symmetric interior Nash equilibrium.

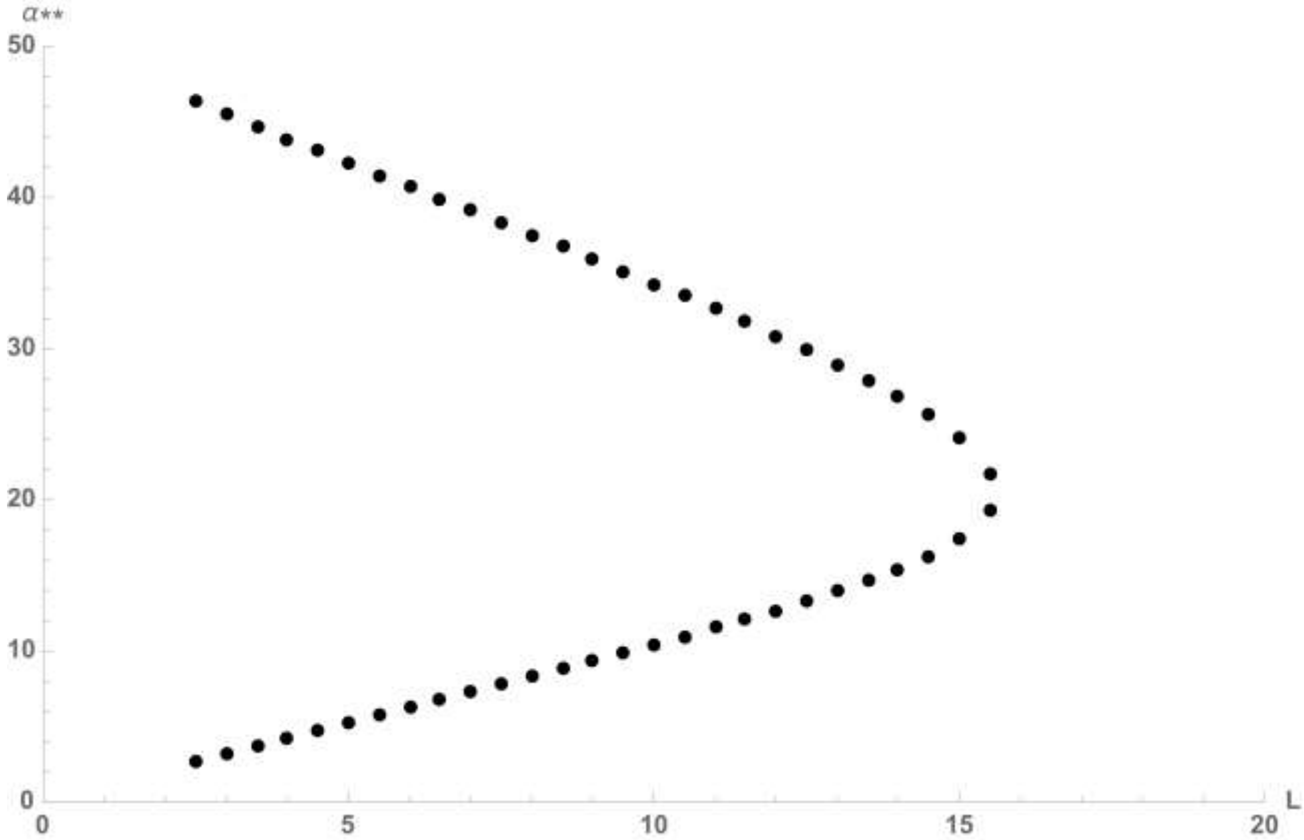
As shown in this figure, both markers have inverted-U shapes in the (L, m_1)-space. The peak of the Nash Equilibrium markers, which represents the maximum value of contribution in Nash equilibria, is $m_1 = 3.66904$ when $L = 36$. If $L < 36$, a further increase in L increases the self-protection contribution in the Nash equilibrium. If $L > 36$, a further increase in L decreases the contribution.

The inverted U-shape is explained as follows. An increase in loss raises both the marginal benefit and the marginal cost of self-protection. On the one hand, an increase in loss reduces the consumption in the bad state, which expands the welfare difference between the two states. Thus, an increase in loss raises the marginal benefit of improving the probability of the good state, and it, therefore, increases the marginal benefit of self-protection. On the other hand, an increase in loss reduces the disposable income in the bad state, which implies that the resource transformed into self-protection becomes increasingly valuable. If a loss in the bad state is not significantly large (when $L < 36$ in Fig. 3), the former effect would dominate. Then, an increase in loss raises the contribution to self-protection. If this loss is significantly large (when $L > 36$), the latter effect would dominate. Then, a further increase in loss reduces the contribution.

Combining Table3 and Figure 2, we could explain how the socially optimal estimation bias induces a country to contribute the socially optimal self-protection as follows. We remember that an overestimation of loss has the same impact on the behavior of a country as an increase in the true loss. Suppose that $L = 10$, such as in Table3. Then, the socially optimal level of contribution is 2.480, which is represented by point *P* in the figure. On the contrary, the contribution in the Nash equilibrium is 1.105. If the country overestimates its loss by 10.429, the overestimated loss becomes 20.429. Then, its contribution increases to 2.480. Although the exact point of this case is not presented in the figure, it could be drawn near point *Q*. Additionally, the country would contribute 2.480 units to the self-protection public good if it overestimates its loss by 34.301. This case is not illustrated in the figure, but the associated point could be illustrated around point *R*. As the true loss in the bad state increases, the optimal self-protection increases, and the points *Q* and *R* converge.

Figure 2 also shows that, if loss in the bad state rises further, the social optimum and the Nash equilibrium tend to converge because, when a loss in the bad state is significantly large, the disposable income in this state becomes so limited that the socially optimal level of self-protection becomes negligible due to the tight budget constraint.

Figure3 Socially optimal estimation bias and the true loss



Source: Authors.

Figure 3 shows how the optimal estimation bias changes for loss in the bad state. In Figure 3, the parameters are set as they are in Table3, except for loss. We assume that losses of countries A and B are identical to L, which increases from 0 to 20. The horizontal axis represents loss, while the vertical axis represents the socially optimal estimation bias, $\alpha^{A**} = \alpha^{B**} = \alpha^{**}$. An optimal value of α does not exist when $L > 15.5$. As shown in Table3, when $2.5 \leq L \leq 15$ there are two values of socially optimal estimation bias for each given level of loss in the bad state. For example, when $L = 10$, the socially optimal estimation biases are 10.429 and 34.301, such as in Table3. The high (low) value of socially optimal bias decreases (increases) with loss. As loss increases, the high and low values converge.

Table4 Numerical simulations of endogenous overestimation

Scenario	1 Social optimum	4 Endogenous overestimation with non-optimal lobbying cost	5 Endogenous overestimation with optimal lobbying cost
Y^A	50	50	50
Y^B	50	50	50
$\bar{L}^A$	10	10	10
$\bar{L}^B$	10	10	10
α^A	n/a	1.136	10.429
α^B	n/a	1.136	10.429
β	n/a	0.1	0.1
p_0	0.25	0.25	0.25
p_e	1	1	1
θ	0.9	0.9	0.9
c	n/a	0.015	0.011
m_1^A	2.480	1.280	2.480
m_1^B	2.480	1.280	2.480
M_1	4.960	2.559	4.960
$\tilde{W}^A$	n/a	14.519	14.354
$\tilde{W}^B$	n/a	14.519	14.354
W^A	14.559	14.545	14.559
W^B	14.559	14.545	14.559
$\beta(m_1^A + m_1^B) - c(\alpha^A + \alpha^B)$	n/a	0.222	0.261

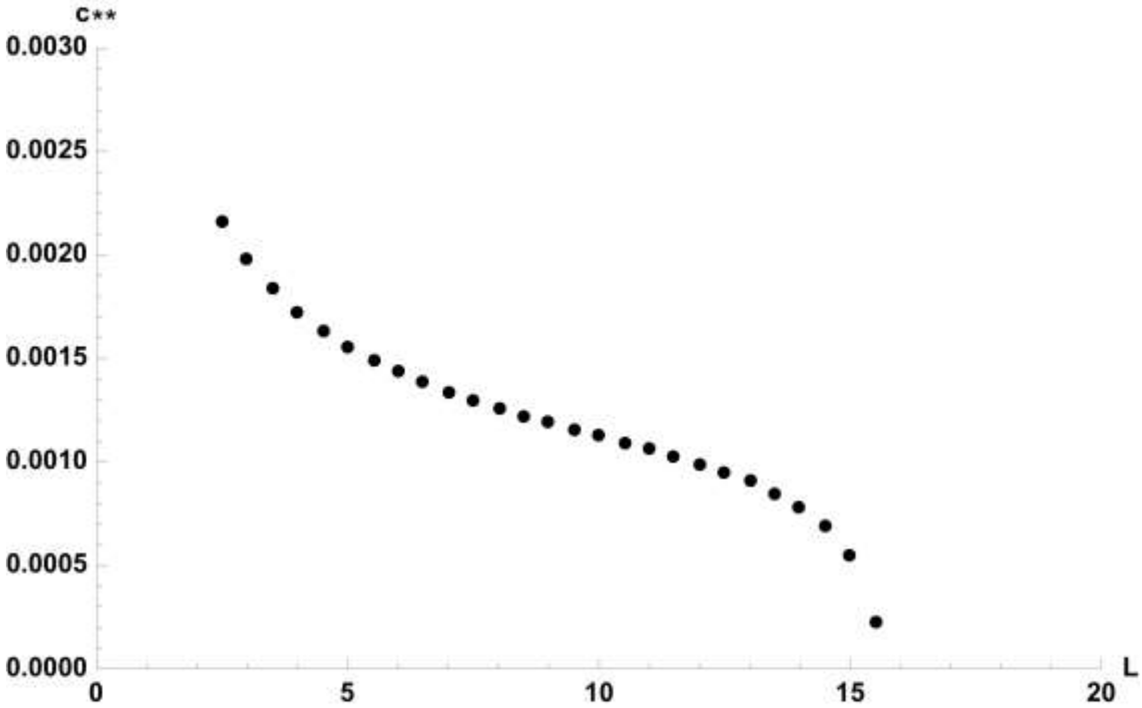
Source: Authors.

C.3 Socially Optimal Unit Cost of Lobbying

Finally, let us investigate the relationship between the optimal unit cost of the NGO's lobbying and the true magnitude of the loss, which corresponds to the severity of the threat to the alliance.

Table4 summarizes the results of numerical simulations of our three-period game in section 4 wherein the NGO endogenously determines the bias in the governments' estimations. For comparison, the first column of the table represents the social optimum, which is identical to that in Table3. The second column represents the result of numerical simulation wherein the international NGO chooses the degree of bias to maximize its net benefit. In this column, the unit cost of lobbying is 0.015. In Period 0, the NGO chooses its lobbying activities to make each government estimate a loss in the bad state more than the true loss by 1.136 units. In Period 1, the government of country A purchases 1.280 units of self-protection. Since country B purchases the same amount of self-protection, the provision of the self-protection public good is 2.559 units. Then, the true expected welfare of each country is 14.545, which is lower than the socially optimum level; however, it is still higher than the Nash equilibrium level in the second column of Table3. Furthermore, the purchase of self-protection in the second column is smaller than that in the first column. Hence, self-protection is still under-provided. The third column of Table4 reports the numerical simulation wherein the marginal cost of lobbying is sufficiently low, such that the NGO chooses the socially optimal estimation bias. Then, the allied countries contribute the socially optimal amounts to self-protection. As shown in the last row of the third column, the net benefit of the NGO is more than that in the second column.

Figure 4 Optimal unit cost of lobbying, c^{**}, and true loss in the bad state



Source: Author.

Figure 4 illustrates how the optimal unit cost of lobbying varies with the true loss in the bad state. The exogenous parameters are set such as in Table4, except for loss in the bad state. The optimal unit cost declines with the value of L, from 2.5 to 15.5. If $L \leq 2$ or $L \geq 16$, the optimal unit cost does not exist, but for different reasons: if $L \leq 2$, the socially optimal self-protection is 0, and the Nash equilibrium self-protection is also 0. Thus, the socially optimal provision of self-provision, which is zero provision, is achieved without any bias. If $L \geq 16$, the socially optimal amount of self-protection is always higher than the maximum value of the Nash equilibrium contribution with estimation bias. Thus, any bias in estimation cannot attain the social optimal, and neither can any unit cost of lobbying.

This figure implies that for a moderately large loss, the optimal unit cost of lobbying should decrease with this loss. Thus, we should be more open to NGOs when the severity of a threat for allied countries increases. However, if the threat is extremely severe, the normative role of NGOs is limited, that is, misestimations cannot attain the social optimum. Even in this case, the NGOs could alleviate the free-riding behavior of allied countries to some extent.

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