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Abstract

We estimate a discrete-time version of Vayanos and Vila's (2009) preferred habitat model, using Japanese government bond yield data. The estimated results indicate that bond excess returns become more sensitive to supply factors in the absence of a zero lower bound constraint unless arbitrageurs become willing to take on more risk.

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1 Introduction

Amid major central banks' purchases of longer-term government bonds under unconventional monetary policy with possible bond sales at the exit stage, there is growing interest in the role of supply and the maturity structure of government bonds in their yields and excess returns. How are supply factors related to yields and excess returns in a zero-lower bound (ZLB) environment compared with a normal environment?

This question is of particular importance for Japan, a country in which government debt is well above 200 percent of GDP—mostly in the form of Japanese government bonds (JGBs)—with potentially high vulnerability to large yield movements in the JGB markets. In Japan, a "preferred habitat" motive (Vayanos and Vila, 2009) in the JGB markets appears to be strong for the following reasons: (i) the Bank of Japan (BoJ) has increased the duration of its long-term JGB purchases under its quantitative-qualitative easing (QQE) since April 2013; (ii) due to the increased and large presence of insurance companies and pension funds in the JGB markets,[1] supply factors can affect yield curves regardless of monetary policy.

To date, relatively few studies, which are predominantly empirical, have examined the effect of supply and maturity structure of JGBs on bond yields and expected returns. Oda and Ueda (2007), an earlier study that analyzes up to the quantitative easing period of 2001-2006, find that the BoJ's purchases of JGBs have no statistically significant effect on term premium.[2] More recent studies after the introduction of the QQE emphasize more on the effect of maturity structure of JGBs on bond yields—these studies generally find positive bond-supply effects on long-term interest rates but differ in the ways that they construct bond supply measures and in their empirical specifications. Iwata (2014) applies the regression approach of Chadha et al. (2013) to estimate

[1] Related statistics and figures are extensively discussed, for example, in Fukunaga, Kato, and Koeda (2015).

[2] During the quantitative easing period from 2001 to 2006, the BoJ purchased JGBs (and other non-monetary assets) to smoothly meet its current-account balance operating target. These JGB purchases, however, did not have an explicit duration target. The average maturity of JGBs held by the BoJ had actually declined during this period.

the supply effect on five-year forward ten-year yield. He uses a measure of the average maturity of JGBs held outside the BoJ as the bond supply measure. Fukunaga, Kato, and Koeda (2015) estimate supply-factor effects on term spreads using two types of empirical specifications: single-equation regression and model-based specifications, which are related to Greenwood and Vayanos (2014)[3] and Hamilton and Wu (2012, henceforth HW), respectively. Furthermore, they take into account insurance companies and pension funds in addition to the BoJ as preferred-habitat investors and construct the corresponding supply measures.

Recently, following the seminal work of Vayanos and Vila (2009),[4] there is a development of term-structure models with preferred-habitat investors and arbitrageurs where supply factors are explicitly modeled in an arbitrage-free framework. The existing models vary in how preferred habitat investors' bond supply or demand is modeled. HW assume that preferred-habitat investors' specific maturity of bond supply depends linearly on the corresponding bond yield; Greenwood and Vayanos (2014) assume that preferred-habitat investors' demand is driven by a stochastic demand factor that follows the Ornstein–Uhlenbeck process; Fan, Li, and Zhou (2013), seeking to explain Chinese bond markets, assume that preferred-habitat investors' demand depends on official lending rates. Li and Wei (2013) introduce preferred-habitat investors' supply factors as additional yield-curve factors with the restrictions that conventional yield-curve factors do not depend on past supply factors, and vice versa. They also introduce an agency MBS supply factor, in addition to a Treasury supply factor, to examine the effects of the Federal Reserve's large-scale asset purchase programs. Kaminska, Vayanos, and Zinna (2011) identify foreign central banks as the primary preferred habitat for the US investors.

We estimate a discrete-time version of Vayanos and Vila's (2009) preferred habitat term-structure model using Japanese government bond data. Our model is based on HW because their model of two regimes—the normal regime (in the absence of the

[3]Funayama (2014) applies the regression approach of Greenwood and Vayanos (2014) using the Ministry of Finance's net marketable bond issuance as a supply measure.

[4]For more discussion of Vayanos and Vila (2009) in the context of unconventional monetary policy, see for example, Joyce, Miles, Scott, and Vayanos (2012).

ZLB constraint) and the zero lower bound regime (at the ZLB)—can address differences between yield curve properties at the ZLB and those in normal times in Japan in a tractable way.[5] We extend HW's model by (i) allowing the coefficients in yield-curve factor dynamics as well as the prices of risk to change at the ZLB to allow greater model flexibility addressing differences in yield curve properties across regimes, (ii) using latent factors instead of observable factors to improve model fit to the data, (iii) writing out arbitrageurs' portfolio optimization problem at the ZLB solving for the bond market equilibrium price, and (iv) performing estimations using Japanese government bond data.

We then examine how much risk premiums must adjust for arbitrageurs to fully absorb preferred-habitat investors' bond supply shifts under each regime. As a benchmark, we assume that the required adjustment for arbitrageurs is to sell long-term (say, 10- or 20-years) bonds and buy short-term (say, 1-year) bonds by one percent share of their total holdings under the ZLB regime, while we consider the opposite transaction under the normal regime. The estimated results indicate that if the degree of arbitrageurs' risk aversion were the same as that calibrated for the US by HW[6], bond risk premium (one-month holding period excess returns at an annualized rate[7]) would decrease by 1.5 basis points for 10-year bonds and 6 basis points for 20-year bonds, with little change in the short-term bond risk premiums under the ZLB regime; on the other hand, they would increase by 7 basis points and 24 basis points respectively under the normal regime. The higher sensitivity of bond risk premiums to supply factors in the normal regime stems from the higher volatilities of yield-curve factors and the greater responsiveness of yields to yield-curve factors. The model-implied risk-premium changes, however, multiply with

[5]Their model with regime shifts addresses nonlinearlity that arises at a ZLB within the tractable affine framework. For discussion beyond the affine framework on the performance of different families of term structure models that enforce a ZLB, see for example, Singleton and Kim (2012) and Ichiue and Ueno (2015).

[6]It appears that the degree of arbitrageurs' risk aversion for JGB markets can take a similar or even higher value than HW's calibrated value (see for e.g., Fukunaga, Kato and Koeda (2015)), even though we use lower frequency data than HW (monthly rather than weekly data) to estimate our model.

[7]Excess returns are the standard measure for bond risk premiums in the term-structure literature (Cochrane and Piazzeti, 2005).

the degree of arbitrageurs' risk aversion, which appears to swing over time in Japan.

Section 2 describes the model. Section 3 discusses the Japanese authorities' (the Ministry of Finance (MoF) and the BoJ) bond-by-bond holding data and how the data is related to the net bond demand of arbitrageurs. Section 4 explains the estimation strategy used and describes estimated results. Section 5 concludes.

2 The model

Following Vayanos and Vila (2009), we assume that there are two types of agents in the government bond markets: preferred-habitat investors and arbitrageurs. Arbitrageurs maximize the mean-variance expected returns on their government bond portfolio. Preferred-habitat investors prefer to hold particular maturities of government bonds. The bond market equilibrium price is determined by equating arbitrageurs' net demand and preferred habitat investors' net supply for different maturities of bonds.

Specifically, our model extends HW by (i) allowing the coefficients in yield-curve factor dynamics as well as the prices of risk to change at the ZLB to allow greater model flexibility. For notational consistency, we put a superscript of "1" on the normal-time model coefficients and a superscript of "0" on the ZLB regime model coefficients, except for transition probabilities; (ii) using latent factors instead of observable factors to improve model fit to the data; and (iii) writing out arbitrageurs' portfolio optimization problem at the ZLB solving for the bond market equilibrium price.

2.1 The set up and normal-time bond pricing

Bond pricing in "normal" times (in the absence of the ZLB) follows the same specification as in HW. HW define the arbitrageurs' rate of return from period t to period $t+1$ on their portfolio ($r_{t,t+1}$) by

$$r_{t,t+1} = \sum_{n=1}^{N} z_{nt} r_{n,t,t+1}, \tag{1}$$

where z_{nt} is the fraction of their portfolio in bonds of maturity n and $r_{n,t,t+1}$ is the holding-period return from period t to $t+1$ on n-period bonds. Because arbitrageurs are assumed to care only about the mean and variance of $r_{t,t+1}$, their optimization problem is given by

$$\max_{z_{1t},\ldots,z_{Nt}} \{E(r_{t,t+1}|\mathbf{f}_t) - (\gamma/2)\, Var(r_{t,t+1}|\mathbf{f}_t)\}, \text{s.t.} \sum_{n=1}^{N} z_{nt} = 1, \tag{2}$$

where γ captures arbitrageurs' risk aversion and $\mathbf{f}_t$ is the 3×1 vector of pricing factors assumed to follow a VAR (1) process with normalization that gives the identity matrix Σ^1 and a 3×1 vector of zeros $\mathbf{c}^1$.

$$\mathbf{f}_{t+1} = \mathbf{c}^1 + \mathbf{\Phi}^1\mathbf{f}_t + \mathbf{\Sigma}^1\mathbf{u}_{t+1},$$

The preferred habitat's net supply of bonds relative to arbitrageurs' net wealth (x_{nt}) is assumed to be

$$x_{nt} \equiv \xi^1_{t,n} - \alpha^1_n y_{t,n}, \quad \text{for } n = 1, ...N, \text{ where } \; \xi^1_{t,n} = \xi^1_n + \boldsymbol{\theta}^{1\prime}_n\mathbf{f}_t, \tag{3}$$

where $y_{t,n}$ is the log yield for n-period bond in period t. The risk-free one period rate ($y_{t,1}$) is assumed to follow an affine function of the yield-curve factors

$$y^1_{t,1} = \delta^1_0 + \boldsymbol{\delta}^1_1\mathbf{f}_t.$$

HW show that at the bond market equilibrium ($z_{nt} = x_{nt}$ for $n = 1, ..., N$), the n-period bond price approximately follows the standard affine term-structure model, $P^1_{t,n} = \exp\left(\bar{a}^1_n + \bar{\mathbf{b}}^1_n\mathbf{f}_t\right)$, where $\bar{a}^1_n$ and $\bar{\mathbf{b}}^1_n$ are yield curve coefficients that have the usual recursive structure.

2.2 Bond pricing at the ZLB

Arbitrageurs now face two types of regimes: the ZLB regime where the ZLB binds and the normal regime where the ZLB does not bind (denoted by $s = 0$ and $s = 1$ respectively). They maximize mean-variance expected returns weighted by the transition probability that the ZLB will continue to bind in the next period (π_{00}) or that it will be lifted (π_{10}). These transition probabilities (π_{i0} for $i = 0, 1$) are assumed to be exogenous and constant and add up to 1 ($\sum_{i=0,1} \pi_{i0} = 1$). The arbitrageurs' optimization problem at the ZLB can be given as

$$\max_{z_{1t},\dots,z_{Nt}} \sum_{i=0,1} \pi_{i0} \left[E\left(r_{t,t+1}|s_{t+1}=i, s_t=0, \mathbf{f}_t\right) - (\gamma/2)\, Var\left(r_{t,t+1}|s_{t+1}=i, s_t=0, \mathbf{f}_t\right)\right], \tag{4}$$

subject to

$$\sum_{n=1}^{N} z_{nt} = 1, \tag{5}$$

where $\mathbf{f}_t$ are the pricing factors assumed to follow a VAR (1) process.

$$\mathbf{f}_{t+1} = \mathbf{c}^i + \mathbf{\Phi}^i \mathbf{f}_t + \mathbf{\Sigma}^i \mathbf{u}_{t+1}. \tag{6}$$

For parsimonious purpose, we assume that Σ^0 is a diagonal matrix. Using the FOCs of the above optimization problem, we can approximately solve for the arbitrageurs' demand equation, $\mathbf{z}_t = (z_{2t}, ..., z_{Nt})'$ (see Appendix A for derivation). $\mathbf{z}_t$ depends on expected excess one-period holding period returns on different maturities of bonds. Given $\mathbf{z}_t$, the arbitrageurs' demand for short-term bonds (z_{1t}) can be derived from eq. (5).

The preferred habitat's net supply of bonds (x_{nt}) is modeled in the same manner as the normal-time model except that the supply-equation coefficients are allowed to take different values at the ZLB. The risk-free one period rate (y_{1t}) is also modeled in the same manner as the normal-time model except that we impose zero restrictions on the coefficients for factors $(\boldsymbol{\delta}_1^0)$ as in HW,

$$y_{t,1}^0 = \delta_0^0 + \boldsymbol{\delta}_1^0 \mathbf{f}_t, \;\; \delta_0^0 \simeq 0 \text{ and } \boldsymbol{\delta}_1^0 = [0, 0, 0].$$

At the bond market equilibrium, the arbitrageurs' net demand should equal the preferred habitats' net supply. Appendix A shows that by equating the demand and supply functions for the n-period bond, the equilibrium bond prices ($P_{t,n}^0$ for $n = 1, ..., N,$) can be derived as follows.

$$p_{t,n}^0 = \bar{a}_n^0 + \bar{\mathbf{b}}_n^0 \mathbf{f}_t, \tag{7}$$

$$\bar{a}_n^0 = \bar{a}_1^0 + \sum_{i=0,1} \pi_{i0} \left[\bar{a}_{n-1}^i + \bar{\mathbf{b}}_{n-1}^i \underbrace{\left\{\mathbf{c}^i - \mathbf{\Sigma}^i \boldsymbol{\lambda}^i\right\}}_{\equiv \mathbf{c}^Q} + (1/2)\, \bar{\mathbf{b}}_{n-1}^i \mathbf{\Sigma}^i \mathbf{\Sigma}^{i\prime} \bar{\mathbf{b}}_{n-1}^{i\prime} \right], \tag{8}$$

$$\bar{\mathbf{b}}_n^0 = \sum_{i=0,1} \pi_{i0} \bar{\mathbf{b}}_{n-1}^i \underbrace{\left\{\mathbf{\Phi}^i - \mathbf{\Sigma}^i \mathbf{\Lambda}^i\right\}}_{\equiv \mathbf{\Phi}^Q} + \bar{\mathbf{b}}_1^0, \tag{9}$$

where $p^0_{t,n} = \ln\left(P^0_{t,n}\right)$, $\bar{a}^0_n$ and $\bar{\mathbf{b}}^0_n$ are yield curve coefficients at the ZLB. $\mathbf{c}^Q$ and $\mathbf{\Phi}^Q$ are factor-dynamics coefficients under the risk-neutral ($\mathbf{Q}$) measure, and are, as in HW, assumed to be the same across regimes. The prices of risk coefficients are expressed as

$$\boldsymbol{\lambda}^i = \gamma \mathbf{\Sigma}^{i\prime} \sum_{n=2}^{N} \bar{\mathbf{b}}^{i\prime}_{n-1} \left(\xi^0_n + \left(\alpha^0_n/n\right) \bar{a}^0_n\right), \; \mathbf{\Lambda}^i = \gamma \mathbf{\Sigma}^{i\prime} \sum_{n=2}^{N} \bar{\mathbf{b}}^{i\prime}_{n-1} \left(\vartheta^0_n + \left(\alpha^0_n/n\right) \bar{\mathbf{b}}^0_n\right).$$

Thus, in equilibrium, bond prices at the ZLB follow the standard affine term structure model with regime shifts. Furthermore, it can be shown (see Appendix A) that the prices of risk can be expressed as a function of $\mathbf{z}_t$,

$$\boldsymbol{\lambda}^i_t \equiv \boldsymbol{\lambda}^i + \mathbf{\Lambda}^i \mathbf{f}_t = \gamma \mathbf{\Sigma}^{i\prime} \sum_{j=2}^{N} z_{jt} \bar{\mathbf{b}}^{i\prime}_{j-1}. \tag{10}$$

2.3 Risk premium

At the bond market equilibrium, the model-implied one-period holding-period excess log return under each regime can be expressed as follows. Under the normal regime

$$E_t\left(rx^1_{t+1,n}\right) \equiv E_t\left(p^1_{t+1,n-1}\right) - p^1_{t,n} - y^1_{1,t} = C^1 + \gamma \bar{\mathbf{b}}^1_{n-1} \mathbf{\Sigma}^1 \mathbf{\Sigma}^{1\prime} \sum_{j=2}^{N} z_{jt} \bar{\mathbf{b}}^{1\prime}_{j-1}. \tag{11}$$

Under the ZLB regime,

$$E_t\left(rx^0_{t+1,n}\right) \equiv \left(\sum_{i=0,1} \pi_{i0} p^i_{t+1,n-1}\right) - p^0_{t,n} - y^0_{1,t} = C^0 + \gamma \sum_{i=0,1} \pi_{i0} \bar{\mathbf{b}}^i_{n-1} \mathbf{\Sigma}^i \mathbf{\Sigma}^{i\prime} \sum_{j=2}^{N} z_{jt} \bar{\mathbf{b}}^{i\prime}_{j-1}. \tag{12}$$

C^1 and C^0 are constant.[8] The second terms on the RHS in eqs. (11) - (12) can be further rewritten as an affine function of yield-curve factors by using eq. (3) and the market clearing condition.

3 Bond-by-bond data for z_{nt}

This section discusses data for the net bond demand of arbitrageurs (z_{nt}).[9] Because z_{nt} equals to the net bond supply of preferred habitat investors at the bond market equilib-

[8] with $C^1 = -(1/2)\, \bar{\mathbf{b}}^1_{n-1} \mathbf{\Sigma}^1 \mathbf{\Sigma}^{1\prime} \bar{\mathbf{b}}^{1\prime}_{n-1}$ and $C^0 = -\sum_{i=0,1} \pi_{i0} \,(1/2)\, \bar{\mathbf{b}}^i_{n-1} \mathbf{\Sigma}^i \mathbf{\Sigma}^{i\prime} \bar{\mathbf{b}}^{i\prime}_{n-1}$.

[9] Note that such data is, however, not needed for the model estimation per se (see Section 4.1 for estimation strategy)

rium, we may construct data for z_{nt} from the supply side, specifically by constructing data on 1) preferred habitat investors' net bond supply and 2) arbitrageurs wealth, and dividing 1) by 2).

Suppose preferred habitat investors in Japan consist of only fiscal and monetary authorities (i.e., the MoF as the issuer of JGBs and the BoJ under unconventional monetary policies). This assumption is consistent with Iwata (2014) because he uses the average maturity of JGBs held outside the BoJ as a bond supply measure. Under this assumption, preferred-habitat investors' net supply variable can be constructed by subtracting the stock of BoJ's bond holdings from that of JGB net market issuance (i.e., initial issues plus reopened issues minus buybacks). This supply variable can be constructed on a bond-by-bond basis at a monthly frequency, using BoJ's bond-by-bond holding data from its official website (available only from June 2001), and using JGB net market issuance data from the *Ko-Shasai-Binran* (Japanese bond handbook) of Japan securities dealer association. We use data on the fixed-rate JGBs, thus excluding data on floating-rate bonds, inflation-linked bonds, and treasury bills (i.e., bonds with less than one year of maturity). We also focus on JGBs financed in the markets excluding directly underwritten bonds. Data on arbitrageurs wealth can be constructed by adding the preferred-habitat investors' net supply over maturity (i.e., $w_t = \sum_{j=1}^{N} z_{jt}$).

Figure 1 reports the BoJ's JGB holdings in billions of yen (Figure 1a) and its share of total net market issuance (Figure 1b) from June 2001 to December 2014, corresponding to each specified range of bond maturities. Since the QQE began in April 2013, the BoJ's holdings, in terms of level, have rapidly increased, notably with respect to maturities between (i) four and five years followed by those between three and four years (Figure 1a, the second chart); (ii) nine and ten years followed by those between eight and nine years (Figure 1a, the third chart); and (iii) ten and twenty years (Figure 1a, the bottom chart). In terms of share, however, only bond purchases with maturities from eight to ten years stand out relative to the quantitative easing period from 2001 to 2006 due to the increased net market issuance in other maturities.

Figure 2 reports the net bond supply of preferred habitat investors from April 2013 to December 2014. We focus on this short period to examine the QQE period and to exclude a period of rapid adjustment in maturity structure by insurance companies.[10] Since the QQE began, the net supply of bonds with maturities between three and five years and between eight and ten years has declined by a few percentage points, whereas the net bond supply with maturities over ten years somewhat has increased.

4 Estimation

4.1 Estimation strategy

For bond yield data, we use Bloomberg's zero yield curve data (end-of-month) for Japan. The sample period for the normal-time model starts after the collapse of the bubble and ends a month before the BoJ started the zero interest rate policy (February 1992–February 1999). The sample period for the term-structure model at the ZLB starts from the latest ZLB regime period identified by Hayashi and Koeda (2014) (December 2008–December 2014). We use bond yields of 3, 24, 60, and 120 month maturities for estimation, allowing the 60-month bond yield equation for measurement error. Table 1 reports summary statistics.

Our model estimation is based on Hamilton and Wu's (2012b) minimum-chi squared estimation and asymptotic standard error calculation method. As in Hamilton and Wu (2012a), we estimate model parameters under each regime separately. We apply their estimation method to the just-identified three latent factor model to estimate the normal-time term structure coefficients ($\mathbf{c}^Q, \mathbf{\Phi}^Q, \bar{a}_1^1, \bar{\mathbf{b}}_1^1, \mathbf{\Phi}^1$), normalizing $\mathbf{\Sigma}^1$and $\mathbf{c}^1$ to a 3×3 identity matrix and a 3×1 zero vector, respectively. We then apply their method to an overidentified model to estimate the ZLB regime term structure coefficients (π_{00}, δ_0^0, $\mathbf{c}^0$, $\mathbf{\Sigma^0}$, $\mathbf{\Phi}^0$) taking the normal-regime model parameters as given. As in HW, model's deep parameters ($\gamma, \alpha_2^i, ..., \alpha_N^i, \xi_2^i, ..., \xi_N$, for $i = 0, 1$) cannot be estimated by this term-

[10]It may be natural to treat insurance companies and pension funds as additional preferred habitat investors. However, their bond-by-bond holding data is not publicly available. Using disclosures information, Fukunaga, Kato, and Koeda (2015) construct bond holding data including insurance companies and pensions as additional preferred habitat investors. See Appendix A of their paper for a detailed description of data construction.

structure model estimation method. Appendix B provides a detailed description of the estimation strategy for the ZLB regime coefficients.

4.2 Estimated results

The average yield curve under each regime (Figure 3a) indicates that the yield curve has flattened on average at the ZLB (consistent with previous findings such as those of Okina and Shiratsuka, 2004, Oda and Ueda, 2007, and Koeda, 2013), and factor loadings under each regime (Figure 3b) indicate that yields became less responsive to yield-curve factors at the ZLB, especially the third yield-curve factor (lower right in Figure 3b).

Table 2 reports estimated parameters. The probability that the ZLB regime continues into the next month (π_{00}) is estimated to be 0.93. The first and second yield-curve factors become much less volatile (compare the (1,1) and (2,2) elements of $\mathbf{\Sigma^1}$ and $\mathbf{\Sigma^0}$) under the ZLB regime, while the third yield-curve factor becomes more volatile under the ZLB regime, although its link with yields notably weakens.

How are supply factors related to excess bond returns under each regime? This question can be examined, given the estimated model parameters and the degree of arbitrageurs' risk aversion (γ), using eqs. (11) - (12). For the benchmark case, we assume that arbitrageurs are required to sell long-term (say 10- or 20-year) bonds and buy shorter-term (say 1- or 5-year) bonds by one percent share of their total bond holdings under the ZLB regime, whereas they are required to perform the opposite transactions under the normal regime. The corresponding changes in expected risk premium (one-period holding period excess returns) for n-period bond can be computed by

$$\Delta E_t \left(\widehat{rx^1_{t+1,n}}\right) = \tilde{\gamma}\widehat{\bar{\mathbf{b}}^1_{n-1}}\hat{\mathbf{\Sigma}}^1\hat{\mathbf{\Sigma}}^{1\prime}\left(-0.01\widehat{\bar{\mathbf{b}}^{1\prime}_{short-1}} + 0.01\widehat{\bar{\mathbf{b}}^{1\prime}_{long-1}}\right), \quad (13)$$

$$\Delta E_t \left(\widehat{rx^0_{t+1,n}}\right) = \tilde{\gamma}\sum_{i=0,1}\hat{\pi}_{i0}\widehat{\bar{\mathbf{b}}^i_{n-1}}\hat{\mathbf{\Sigma}}^i\hat{\mathbf{\Sigma}}^{i\prime}\left(0.01\widehat{\bar{\mathbf{b}}^{1\prime}_{short-1}} - 0.01\widehat{\bar{\mathbf{b}}^{1\prime}_{long-1}}\right), \quad (14)$$

where $long$ = 120 or 240, $short$ = 12 or 60, and n = $short$ or $long$. Parameters denoted by "hat" are estimated values while $\tilde{\gamma}$ denotes a candidate value. Because γ is a deep parameter that cannot be estimated via the estimation procedure described in the previous section, we have attempted to estimate γ using either the prices-of-risk equation (eq. (10)) or the risk premium equations (eqs. (11)-(12)), replacing the model parameters with estimated values and z_{nt} with data described in Section 3. However, the estimated γ turns out to be highly time varying under either method.

We thus report results with different values of γ. Table 3 reports model-implied risk premium changes in response to the supply shift in the benchmark case for $\gamma = 50$, 100, 200 and 500. For example, if $\gamma = 100$ (which corresponds to HW's calibrated value for the United States), $short = 12$, and $long = 120$, the 10-year bond risk premium is expected to increase by 7 basis points under the normal regime (by eq. (13)), while it is expected to decrease by only about 2 basis points under the ZLB regime (by eq. (14)). On the other hand, the corresponding risk-premium changes for one-year bonds are small (see "1-year" columns in Table 3), implying that the effect of BoJ's long-term bond purchases that involves reserve accumulation should be similar to that of its maturity swaps that involve no reserve changes.

Not surprisingly, the supply effect on risk premium intensifies with maturity. If the long-term bonds were 20-year bonds instead of 10-year bonds (i.e., set $long = 240$ instead of 120), then the 20-year bond risk premium is expected to increase by 24 basis points under the normal regime and it is expected to decrease by about 6 basis points under the ZLB regime. Thus the supply effect of 20-year bonds is three or four times larger than that of 10-year bonds.

The higher sensitivity of long-term bond excess returns to supply factors in the normal regime stems from the higher volatilities of yield-curve factors ($\mathbf{\Sigma^1}$ compared with $\mathbf{\Sigma^0}$) as well as the greater responsiveness of yields to yield-curve factors (Figure 3b). These risk premium changes multiply with the value of γ. If γ doubles to 200, changes under the normal regime double to 15 basis points for 10-year bonds and 48 basis points for 20-year bonds. This implies that the degree of arbitrageurs' risk aversion, which could take a much higher value than 100 in Japan under some circumstances,[11] is a key determinant of the sensitivity of bond yields or excess returns to supply factors.

[11] For more discussion see Fukunaga, Kato and Koeda (2015). They attempt to estimate this deep parameter (γ) and find that the estimated value is sensitive to both time and bond maturity.

5 Conclusion

This paper finds that bond excess returns can be more sensitive to supply factors in normal times unless arbitrageurs become willing to take on more risks. Looking ahead, there is much concern about the impact that exit from the current unconventional monetary policy will have on JGB markets. How much supply factors affect bond yields and bond risk premia at the exit depends on how quickly arbitrageurs can adjust their risk appetite to a more volatile market environment. If arbitrageurs cannot favor volatility in the normalization process—for example, under new stricter regulation of interest rate risk—then the yield-curve steepening effect of bond sales at the exit by BoJ could easily outweigh the flattening effect of its bond purchases at the ZLB. Furthermore, the choice of bond maturity sold at the exit seems to be important. For example, the estimated results imply that doubling the maturity from 10 to 20 years would more than triple the supply effect on risk premium.

Going forward, the existing term structure model with preferred habitat could be further developed in several ways. First, existing models usually assume that arbitrageurs' risk aversion is constant over time. Given the weak empirical support for this assumption, arbitrageurs' risk aversion could be allowed to be time varying. Second, the preferred habitat investors' supply equation may explicitly take into account different types of investors to analyze each of their roles in the bond markets.

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A Approximated term structure model at the ZLB

This appendix solves for the equilibrium bond prices by equating the preferred habitat investors' net supply function and the arbitrageurs' demand function. As mentioned in Section 2.2, the net supply equation for the preferred habitat investors is assumed to be

$$x_{nt} \equiv \xi_{t,n}^{0} - \alpha_n^0 y_{t,n}, \text{ for } n = 1, ...N, \; \xi_{t,n} = \xi_n^0 + \vartheta_n^0 \mathbf{f}_t. \tag{15}$$

The arbitrageurs' demand function can be derived by solving their optimization problem (i.e., eq. (4), subject to eqs. (5)). The corresponding FOCs are given by

$$z_{1t} \; : \; r_{1,t,t+1} = \phi, \tag{16}$$

$$z_{nt} \; : \; \sum_{i=0,1} \pi_{i0} \begin{bmatrix} \frac{\partial}{\partial z_{nt}} E\left(r_{t,t+1} | s_{t+1} = i, s_t = 0, \mathbf{f}_t\right) \\ -\frac{\gamma}{2} \frac{\partial}{\partial z_{nt}} Var\left(r_{t,t+1} | s_{t+1} = i, s_t = 0, \mathbf{f}_t\right) \end{bmatrix} = \phi, \quad n = 2, ..., N \tag{17}$$

where ϕ is the Lagrange multiplier and the arbitrageurs' rate of return on their portfolio ($r_{t,t+1}$), which is defined by eq. (1), is the sum of holding-period returns on an n-period bond ($r_{n,t,t+1}$) weighted by the fraction of their portfolio in the bond of maturity n (z_{nt}).

We derive the net demand function of arbitrageurs given the conjecture that the zero-coupon bond price can be expressed as an exponential affine function of yield-curve factors ($P_{t,n}^{s_t} = \exp\left(\bar{a}_n^{s_t} + \bar{\mathbf{b}}_n^{s_t} \mathbf{f}_t\right)$). Using an approximation of $\exp(x) - 1 \simeq x$, the portfolio mean return and variance ($E_t\left(r_{t,t+1}\right)$ and $V_t\left(r_{t,t+1}\right)$) can be approximated by

$$\begin{aligned} E\left(r_{t,t+1} | s_{t+1} = i, s_t = 0, \mathbf{f}_t\right) &\simeq -z_{1t}\left(\bar{a}_1^0 + \bar{\mathbf{b}}_1^0 \mathbf{f}_t\right) \\ &\quad + \sum_{n=2}^{N} z_{nt} \begin{bmatrix} \bar{a}_{n-1}^i + \bar{\mathbf{b}}_{n-1}^i\left(\mathbf{c}^i + \mathbf{\Phi}^i \mathbf{f}_t\right) \\ + (1/2)\, \bar{\mathbf{b}}_{n-1}^i \mathbf{\Sigma}^i \mathbf{\Sigma}^{i\prime} \bar{\mathbf{b}}_{n-1}^{i\prime} - \bar{a}_n^0 - \bar{\mathbf{b}}_n^0 \mathbf{f}_t \end{bmatrix}, \end{aligned} \tag{18}$$

$$V\left(r_{t,t+1} | s_{t+1} = i, s_t = 0, \mathbf{f}_t\right) \simeq \left(\sum_{j=2}^{N} z_{jt} \bar{\mathbf{b}}_{j-1}^i\right) \mathbf{\Sigma}^i \mathbf{\Sigma}^{i\prime} \left(\sum_{j=2}^{N} z_{jt} \bar{\mathbf{b}}_{j-1}^{i\prime}\right). \tag{19}$$

Therefore the two derivatives that appear in eq. (17) can be expressed as

$$\frac{\partial}{\partial z_{nt}} E\left(r_{t,t+1} | s_{t+1} = i, s_t = 0, \mathbf{f}_t\right) \simeq \begin{matrix} \bar{a}_{n-1}^i + \bar{\mathbf{b}}_{n-1}^i\left(\mathbf{c}^i + \mathbf{\Phi}^i \mathbf{f}_t\right) \\ + (1/2)\, \bar{\mathbf{b}}_{n-1}^i \mathbf{\Sigma} \mathbf{\Sigma}' \bar{\mathbf{b}}_{n-1}^{i\prime} - \bar{a}_n^0 - \bar{\mathbf{b}}_n^0 \mathbf{f}_t \end{matrix}, \tag{20}$$

$$\frac{\gamma}{2} \frac{\partial}{\partial z_{nt}} V\left(r_{t,t+1} | s_{t+1} = i, s_t = 0, \mathbf{f}_t\right) \simeq \gamma \bar{\mathbf{b}}_{n-1}^i \mathbf{\Sigma}^i \mathbf{\Sigma}^{i\prime} \sum_{j=2}^{N} z_{jt} \bar{\mathbf{b}}_{j-1}^{i\prime}. \tag{21}$$

Using the above approximated eqs. (20) and (21), the combined FOCs are given by

$$\bar{a}_n^0 + \bar{\mathbf{b}}_n^0 \mathbf{f}_t = \sum_{i=0,1} \pi_{i0} \begin{bmatrix} \bar{a}_{n-1}^i + \bar{\mathbf{b}}_{n-1}^i \left(\mathbf{c}^i + \mathbf{\Phi}^i \mathbf{f}_t\right) \\ + (1/2)\, \bar{\mathbf{b}}_{n-1}^i \mathbf{\Sigma}^i \mathbf{\Sigma}^{i\prime} \bar{\mathbf{b}}_{n-1}^{i\prime} \end{bmatrix} - \sum_{i=0,1} \pi_{i0} \bar{\mathbf{b}}_{n-1}^i \mathbf{\Sigma}^{i\prime} \lambda_t^i + \bar{a}_1^0 + \bar{\mathbf{b}}_1^0 \mathbf{f}_t, \quad (22)$$

where λ_t^i is a function of $\mathbf{z}_t = (z_{2t}, ..., z_{Nt})'$

$$\lambda_t^i \equiv \gamma \mathbf{\Sigma}^{i\prime} \sum_{j=2}^{N} z_{jt} \bar{\mathbf{b}}_{j-1}^{i\prime}. \quad (23)$$

Because eq. (22) holds for all $\mathbf{f}_t$ we can solve for $\mathbf{z}_t$ as

$$\mathbf{z}_t \simeq \Omega^{-1} \left\{ \sum_{i=0,1} \pi_{i0} \begin{bmatrix} \mathbf{A}_{-1}^i + \mathbf{B}_{-1}^i \left(\mathbf{c}^i + \mathbf{\Phi}^i \mathbf{f}_t\right) \\ + (1/2)\, \mathbf{B}_{-1}^i \mathbf{\Sigma}^i \mathbf{\Sigma}^{i\prime} \mathbf{B}_{-1}^{i}{}' \end{bmatrix} - A^0 - \mathbf{B}^0 \mathbf{f}_t + \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \left(\bar{a}_1^0 + \bar{\mathbf{b}}_1^0 \mathbf{f}_t\right) \right\}, \quad (24)$$

where

$$\begin{aligned} \mathbf{A}(0) &= \begin{bmatrix} \bar{a}_2^0 \\ \vdots \\ \bar{a}_N^0 \end{bmatrix}, \ \mathbf{B}(0) = \begin{bmatrix} \bar{\mathbf{b}}_2^0 \\ \vdots \\ \bar{\mathbf{b}}_N^0 \end{bmatrix}, \ \mathbf{A}_{-1}^i = \begin{bmatrix} \bar{a}_1^i \\ \vdots \\ \bar{a}_{N-1}^i \end{bmatrix}, \ \mathbf{B}_{-1}^i = \begin{bmatrix} \bar{\mathbf{b}}_1^i \\ \vdots \\ \bar{\mathbf{b}}_{N-1}^i \end{bmatrix}, \\ \Omega &\equiv \sum_{i=0,1} \pi_{i0} \gamma \mathbf{B}_{-1}^i \mathbf{\Sigma}^i \mathbf{\Sigma}^{i\prime} \left(\mathbf{B}_{-1}^i\right)'. \end{aligned}$$

Eq. (24) is the arbitrageurs' demand equation. Given $\mathbf{z}_t$, the arbitrageurs' demand for the short-term bond, z_{1t}, can be derived by $z_{1t} = 1 - \sum_{n=2}^{N} z_{nt}$.

At the bond market equilibrium, the arbitrageurs' net demand should be equal to the preferred habitats' net supply ($z_{nt} = x_{nt}$ for all n). Equating the demand and supply functions, the equilibrium bond prices satisfy the following equation:

$$\begin{aligned} \bar{a}_n^0 + \bar{\mathbf{b}}_n^0 \mathbf{f}_t &= \sum_{i=0,1} \pi_{i0} \left[\bar{a}_{n-1}^i + \bar{\mathbf{b}}_{n-1}^i \left(\mathbf{c}^i + \mathbf{\Phi}^i \mathbf{f}_t\right) + (1/2)\, \bar{\mathbf{b}}_{n-1}^i \mathbf{\Sigma}^i \mathbf{\Sigma}^{i\prime} \bar{\mathbf{b}}_{n-1}^{i\prime}\right] \\ &\quad - \sum_{i=0,1} \pi_{i0} \bar{\mathbf{b}}_{n-1}^i \mathbf{\Sigma}^{i\prime} \gamma \mathbf{\Sigma}^{i\prime} \sum_{j=2}^{N} \begin{pmatrix} \xi_j^0 + \vartheta_j^0 \mathbf{f}_t \\ + \bar{\mathbf{b}}_j^0 \left(\bar{a}_j^0 + \bar{\mathbf{b}}_j^0 \mathbf{f}_t\right) \end{pmatrix} \bar{\mathbf{b}}_{j-1}^{i\prime} \\ &\quad + \bar{a}_1^0 + \bar{\mathbf{b}}_1^0 \mathbf{f}_t. \quad \text{for } n = 2, ..., N. \end{aligned}$$

The above equation can be rewritten as

$$0 = \left\{ -\bar{a}_n^0 + \bar{a}_1^0 + \sum_{i=0,1} \pi_{i0} \begin{bmatrix} \bar{a}_{n-1}^i + \bar{\mathbf{b}}_{n-1}^i \mathbf{c}^i \\ +(1/2)\,\bar{\mathbf{b}}_{n-1}^i \boldsymbol{\Sigma}^i \boldsymbol{\Sigma}^{i\prime} \bar{\mathbf{b}}_{n-1}^{i\prime} \\ -\bar{\mathbf{b}}_{n-1}^i \boldsymbol{\Sigma}^i \lambda^i \end{bmatrix} \right\} + \left\{ -\bar{\mathbf{b}}_n^0 + \bar{\mathbf{b}}_1^0 + \sum_{i=0,1} \pi_{i0} \left\{ \bar{\mathbf{b}}_{n-1}^i \boldsymbol{\Phi}^i - \bar{\mathbf{b}}_{n-1}^i \boldsymbol{\Sigma}^{i\prime} \boldsymbol{\Lambda}^i \right\} \right\} \mathbf{f}_t, \tag{25}$$

where

$$\boldsymbol{\lambda}^i = \gamma \boldsymbol{\Sigma}^{i\prime} \sum_{j=2}^{N} \bar{\mathbf{b}}_{j-1}^{i\prime} \left(\xi_j^0 + \left(\alpha_j^0 / j \right) \bar{a}_j^0 \right), \ \boldsymbol{\Lambda}^i = \gamma \boldsymbol{\Sigma}^{i\prime} \sum_{j=2}^{N} \bar{\mathbf{b}}_{j-1}^{i\prime} \left(\vartheta_j^0 + \left(\alpha_j^0 / j \right) \bar{\mathbf{b}}_j^0 \right).$$

Because eq. (25) must hold for all $\mathbf{f}_t$, the two expressions in the curly brackets on the RHS must equal zero. Thus, for $n = 2, ..., N$, the following equations must hold:

$$\begin{aligned} \bar{a}_n^0 &= \bar{a}_1^0 + \sum_{i=0,1} \pi_{i0} \left[\bar{a}_{n-1}^i + \bar{\mathbf{b}}_{n-1}^i \left\{ \mathbf{c}^i - \boldsymbol{\Sigma}^{i\prime} \boldsymbol{\lambda}^i \right\} + (1/2)\,\bar{\mathbf{b}}_{n-1}^i \boldsymbol{\Sigma}^i \boldsymbol{\Sigma}^{i\prime} \bar{\mathbf{b}}_{n-1}^{i\prime} \right], \\ \bar{\mathbf{b}}_n^0 &= \bar{\mathbf{b}}_1^0 + \sum_{i=0,1} \pi_{i0} \bar{\mathbf{b}}_{n-1}^i \left\{ \boldsymbol{\Phi}^i - \boldsymbol{\Sigma}^{i\prime} \Lambda^i \right\}. \end{aligned}$$

The above recursions are consistent with the standard affine term structure model with regime shifts.

For $n = 1$, by combining the following two conditions, the one-period bond price can be represented as an affine function of the yield-curve factors (eq. (26)).

$$\begin{aligned} z_{1t} &= x_{1t} = \xi_1^0 + \vartheta_1^0 \mathbf{f}_t - \alpha_1^0 y_{t,1} \text{ (equilibrium condition)} \\ z_{1t} &= 1 - \sum_{n=2}^{N} z_{nt} \text{ (bond portfolio condition)} \end{aligned}$$

$$y_{t,1} = \underbrace{\left(1/\alpha_1^0\right) \left[\xi_1^0 + \sum_{j=2}^{N} \left(\xi_j^0 + \alpha_j^0 \bar{a}_j^0 / j \right) \right] - 1}_{-\bar{a}_1^0} + \underbrace{\left(1/\alpha_1^0\right) \left[\vartheta_1^0 + \sum_{j=2}^{N} \vartheta_j^0 + \left(\alpha_j^0 / j \right) \bar{\mathbf{b}}_j^0 \right]}_{-\bar{\mathbf{b}}_1^0} \mathbf{f}_t. \tag{26}$$

B Estimating the ZLB regime model

Define $R_{t,1} \equiv [y_{t,3}, y_{t,24}, y_{t,120}]'$ (3-month, 2-year and 10-year yields) as the 3×1 vector of yields without measurement error and $R_{t,2} \equiv y_{t,60}$ (5-year yield) as the yield with measurement error. Using the factor dynamics equation for the ZLB regime (eq. (6)), the corresponding yield equations can be rewritten as

$$\underset{3\times1}{R_{t,1}} = \left(A_1^0 + B_1^0\mathbf{c}^0 - B_1^0\mathbf{\Phi}^0 A_1^0\right) + B_1^0\mathbf{\Phi}^0 R_{t-1,1} + B_1^0\mathbf{\Sigma}^0\mathbf{u}_t, \tag{27}$$

$$\underset{1\times1}{R_{t,2}} = \left(A_2^0 - B_2^0\left(B_1^0\right)^{-1}A_1^0\right) + B_2^0\left(B_1^0\right)^{-1}R_{t,1} + \Sigma^m\varepsilon_t, \tag{28}$$

where

$$A_1^0 \equiv \begin{bmatrix} -\bar{a}_3^0/3 \\ -\bar{a}_{24}^0/24 \\ -\bar{a}_{120}^0/120 \end{bmatrix},\ B_1^0 \equiv \begin{bmatrix} -\bar{\mathbf{b}}_3^0/3 \\ -\bar{\mathbf{b}}_{24}^0/24 \\ -\bar{\mathbf{b}}_{24}^0/120 \end{bmatrix}$$

$$A_2^0 \equiv -\bar{a}_{60}^0/60,\ B_2^0 \equiv -\bar{\mathbf{b}}_{60}^0/60.$$

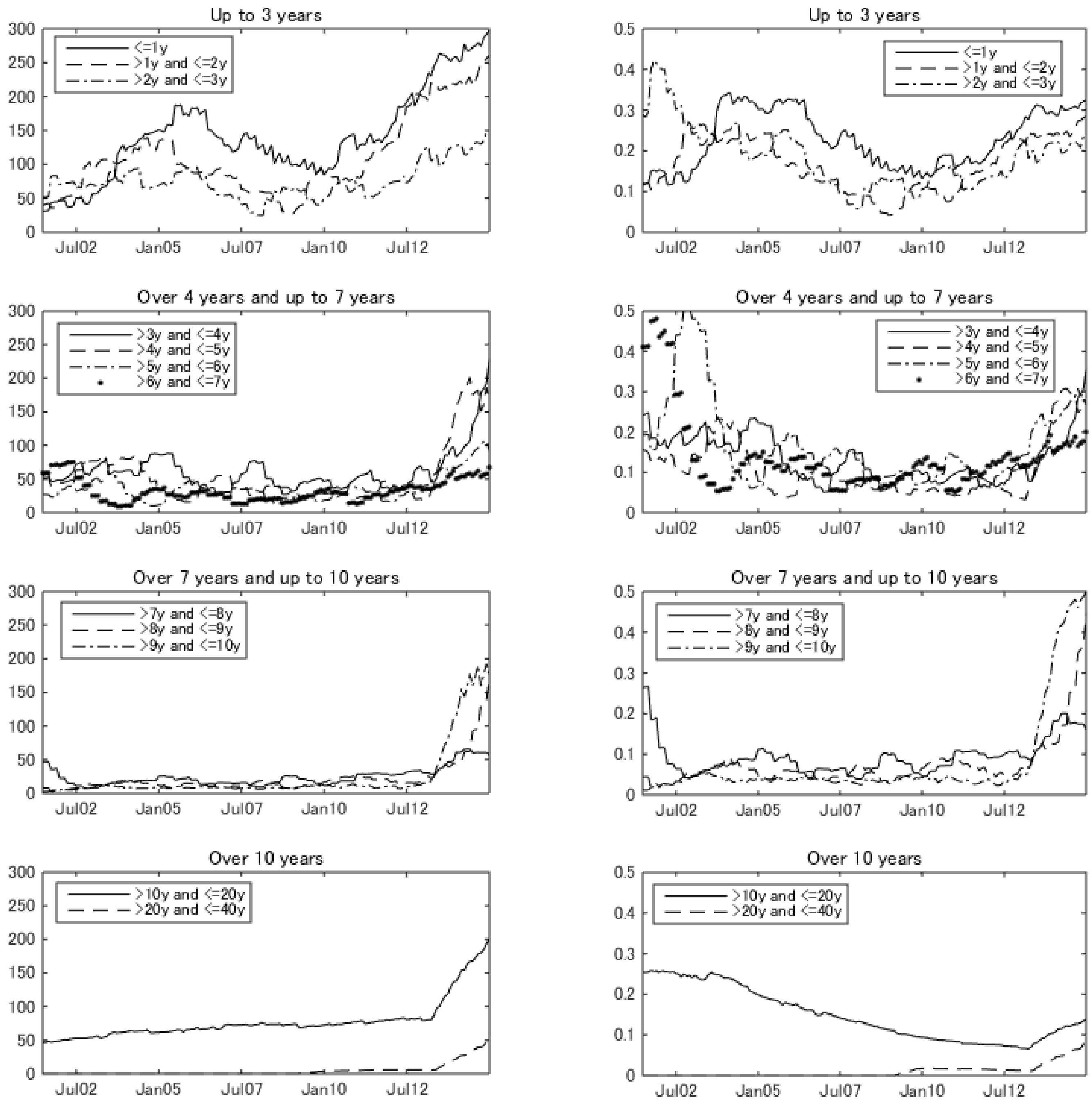
Denote by A_1^+, B_1^+, $\mathbf{\Omega}_1^+$ the OLS coefficients obtained by regressing $R_{t,1}$ on a constant and $R_{t-1,1}$ (corresponding to the constant, $R_{t-1,1}$, and covariance terms respectively) and A_2^+, B_2^+, and Ω_2^+ as the OLS coefficients obtained by regressing $R_{t,2}$ on a constant and $R_{t,1}$ (corresponding to the constant, $R_{t,1}$, and variance terms respectively).

The parameters for the ZLB regime model (π_{00}, δ_0^0, $\mathbf{c}^0$, $\mathbf{\Sigma^0}$, $\mathbf{\Phi}^0$) can be estimated in the following steps. First, given the recursive equations for $\bar{\mathbf{b}}_n^0$ (which are functions only of π_{00} given the normal-regime model parameters and the zero-lower restriction of $\bar{\mathbf{b}}_1^0 = [0,0,0]$, see eq. (9)) estimates of $\mathbf{\Sigma^0}$ and $\mathbf{\Phi}^0$ can be obtained by numerically solving for $\hat{\mathbf{\Sigma}}^\mathbf{0}\hat{\mathbf{\Sigma}}^{\mathbf{0}\prime} = B_1^{-1}\mathbf{\Omega}_1^+ B_1^{-1\prime}$ and $\hat{\mathbf{\Phi}}^0 = B_1^{-1}B_1^+$ (see eq. (27)). Second, given $\hat{\mathbf{\Sigma}}^\mathbf{0}$ and $\hat{\mathbf{\Phi}}^0$, the estimate for $\mathbf{c}^0$ can be obtained by numerically solving for $\hat{\mathbf{c}}^0 = B_1^{-1}\left(A_1^+ - A_1 + B_1\mathbf{\Phi}^0 A_1\right)$ (see eq. (27)). Third, π_{00} can be estimated via the minimum chi-square estimation procedure for an overidentified case, as proposed by Hamilton and Wu (2012b) using eq. (28).

Figure 1. The BoJ's JGB holdings by maturity.

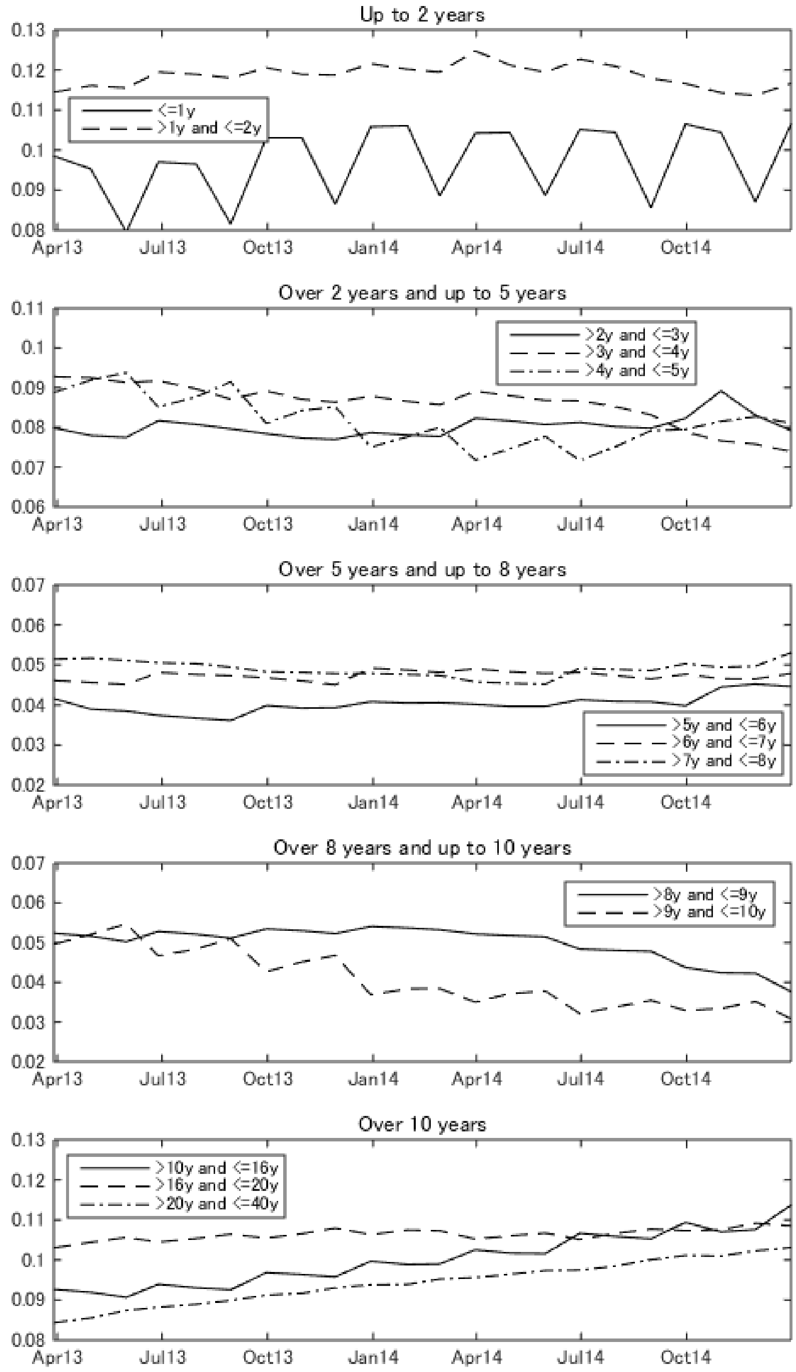
(a) In billions of yen

(b) The share in net market issuance



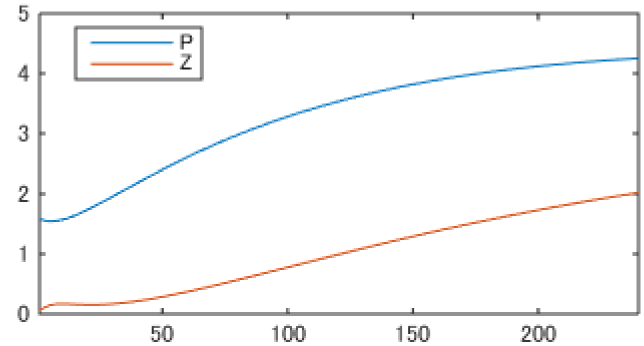
This figure plots BoJ's JGB holdings for specified ranges of maturities (a) in billions of yen and (b) as a fraction of the corresponding maturities of net market MoF's issuance. Net market issuance is MoF's initial plus reopening issues minus buybacks. The sample period is from June 2001 to December 2014.

Figure 2. Preferred-habitat investors' net JGB supply.



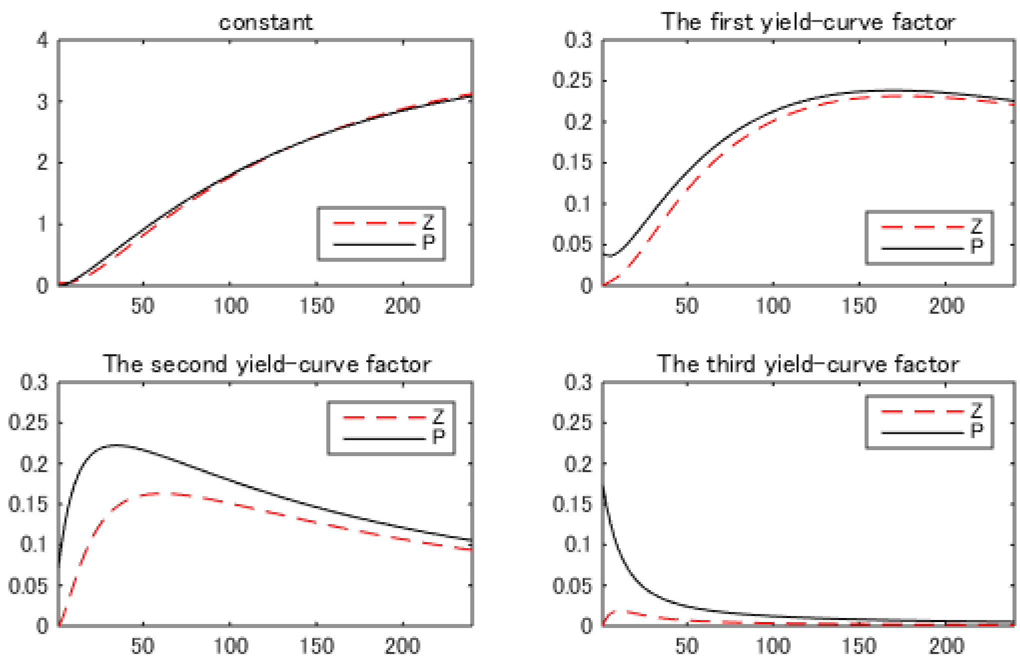
This figure plots the total JGB net market issuance minus the stock of BoJ's bond holdings for each specified range of maturities of bond as a fraction of arbitrageurs' wealth. The sample period is from April 2013 to December 2014.

Figure 3a. Average yield curve under each regime



This figure plots average yield curves, i.e. the model-implied yield curve calculated with the average values of estimated factors under each regime. Z stands for the zero rate regime and P stands for the normal regime.

Figure 3b. Factor loadings under each regime



The upper left, upper right, lower left, and lower right charts correspond to the constant term, the first factor, the second factor, and third factor respectively. Z stands for the zero rate regime and P stands for the normal regime.

Table 1. Summary statistics.

(a) The normal regime period (February 1992--February 1999)

	Central moments				Autocorrelations		
	Mean	Stdev	Skew	Kurt	Lag 1	Lag 2	Lag 3
3-month	1.561	1.418	0.845	2.593	0.991	0.979	0.969
24-month	1.821	1.302	0.592	2.042	0.982	0.963	0.934
60-month	2.678	1.385	0.469	2.124	0.977	0.951	0.914
120-month	3.537	1.300	-0.098	2.024	0.975	0.947	0.914
Excess returns (10y bond)	0.691	2.845	-0.736	4.978	0.045	0.077	-0.274
Excess returns (20y bond)	0.899	5.146	-0.759	5.404	0.050	0.011	-0.283

(b) The zero rate regime period (December 2008--December 2014)

	Central moments				Autocorrelations		
	Mean	Stdev	Skew	Kurt	Lag 1	Lag 2	Lag 3
3-month	0.110	0.059	0.455	4.571	0.953	0.913	0.880
24-month	0.155	0.093	1.210	4.218	0.974	0.934	0.894
60-month	0.371	0.199	0.705	2.574	0.965	0.925	0.880
120-month	0.990	0.315	-0.090	1.819	0.967	0.940	0.910
Excess returns (10y bond)	0.303	0.799	-1.034	5.173	-0.090	0.006	-0.086
Excess returns (20y bond)	0.544	1.767	-1.009	4.865	-0.095	-0.002	-0.100

Risk premium is one-month holding period excess returns. Normal distribution has skewness of zero and kurtosis of 3.

Table 2. Estimated parameters.

Yield curve coefficients / **Transition probability**

$\bar{a}_1^1 \times 1200$	$\bar{\mathbf{b}}_1^1 \times 1200$			$\bar{a}_1^0 \times 1200$	π_{00}
0.00	0.04	0.07	0.17	0.05	0.930
(0.92)	(0.63)	(0.36)	(0.01)	(0.09)	(0.01)

Factor dynamics

$\mathbf{c}^1$	$\mathbf{\Phi}^1$			$\mathbf{\Sigma}^1$		
0.00	0.91	0.10	-0.03	1.00	---	---
---	(1.22)	(1.15)	(0.83)	---	---	---
0.00	0.04	0.78	0.09	---	1.00	---
---	(1.15)	(1.22)	(0.27)	---	---	---
0.00	0.00	0.18	0.85	---	---	1.00
---	(1.58)	(0.07)	(0.05)	---	---	---

$\mathbf{c}^0$	$\mathbf{\Phi}^0$			$\mathbf{\Sigma}^0$		
-0.57	0.97	0.26	0.04	0.45	---	---
(0.32)	(0.06)	(0.16)	(0.16)	(0.013)	---	---
-0.62	-0.13	0.64	-0.01	---	0.30	---
(0.38)	(0.02)	(0.06)	(0.06)	---	(0.002)	---
2.84	0.51	1.00	0.94	---	---	2.06
(0.59)	(0.03)	(0.08)	(0.08)	---	---	(0.007)

$\mathbf{c}^{\mathbf{Q}}$	$\mathbf{\Phi}^{\mathbf{Q}}$			$\mathbf{\Sigma}_{\mathbf{e}}^1 \times 1200$	$\mathbf{\Sigma}_{\mathbf{e}}^0 \times 1200$
0.03	0.99	---	---	0.03	0.12
(1.53)	(0.26)	---	---	(0.002)	(0.009)
0.17	0.03	0.99	---		
(0.23)	(0.01)	(0.26)	---		
-0.04	-0.03	0.20	0.86		
(0.18)	(1.73)	(0.27)	(0.06)		

Values in parenthesis are standard errors. c^1 and $\sum^1$ are a 3×1 vector of zeros and a 3×3 identity matrix.

Table 3. Changes in excess returns in response to supply shifts

	Z		P		Z		P	
γ	10-year	1-year	10-year	1-year	10-year	5-year	10-year	5-year
50	-0.8	-0.02	3.7	0.2	-0.5	-0.2	2.1	0.9
100	-1.5	-0.03	7.4	0.3	-1.0	-0.3	4.3	1.8
200	-3.1	-0.1	14.8	0.7	-1.9	-0.7	8.6	3.5
500	-7.6	-0.2	36.9	1.6	-4.8	-1.7	21.5	8.8

	Z		P		Z		P	
γ	20-year	1-year	20-year	1-year	20-year	5-year	20-year	5-year
50	-2.8	-0.03	12.1	0.2	-2.3	-0.4	9.5	2.0
100	-5.6	-0.05	24.2	0.5	-4.5	-0.8	19.0	4.0
200	11.1	-0.1	48.3	1.0	-9.1	-1.6	38.0	8.0
500	-27.8	-0.3	120.1	2.5	-22.7	-4.1	95.1	19.9

In annualized rate in basis points. This table reports changes in one-month holding period excess returns to a supply shift that requires arbitrageur to sell (buy) the longer-term bonds and buy (sell) the shorter-term bonds by one percent share of their holdings under the zero rate regime (the normal regime) at the bond market equilibrium. We treat 10- or 20-year bonds as "longer-term" and 1- or 5-year bonds as "shorter-term."