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Strategic Managerial Delegation in a Mixed Duopoly with Capacity Choice: Partial Delegation or Full Delegation

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Abstract

This paper analyzes the capacity choice in a mixed duopoly, considering the separation between ownership and management of firms. We introduce the following two alternatives as each firm's delegation type to her/his manager: (*i*) *Partial delegation* – delegating only the quantity setting; (*ii*) *Full delegation* – delegating the determination of both capacity and quantity levels. First, we investigate each firm's capacity scale relative to her/his quantity, given her/his delegation type. Second, we derive the equilibrium delegation type of each firm under the endogenous decision by her/his owner. Finally, we consider the effect of privatization on each firm's delegation type and social welfare.

JEL Classification: D21, L13, L33
Keywords: Mixed Duopoly, Managerial Delegation, Capacity Choice, Privatization

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1 Introduction

This paper examines the issue of capacity choice in a quantity-setting mixed duopoly with homogeneous goods, taking into account the separation between ownership and management of firms. Explicitly considering the power of managers in each firm relative to their owners, we focus on whether or not the owner of each firm delegates to the manager the right to decide capacity scale as well as quantity level.

In the context of private oligopolies, many studies have considered firms' capacity choice issues in several economic environments.[1] Many researchers have already addressed the problem of capacity choice faced by firms in various mixed oligopolistic environments as well.[2] Using the supergame approach as in Davidson and Deneckere (1990), Wen and Sasaki (2001) showed that a public firm can hold excess capacity to sustain a subgame perfect Nash equilibrium. Nishimori and Ogawa (2004) found that in a two-stage game involving capacity choice and quantity setting, a public firm strategically chooses under capacity in a mixed duopoly with homogeneous goods. The result differs different from the fact that each firm chooses excess capacity in most private oligopolistic industries. Lu and Poddar (2005) extended the simultaneous-move competition on both the capacity choice and quantity competition considered in Nishimori and Ogawa (2004) to sequential move cases with respect to the decision of both capacity scale and output level of each firm and found that the public firm never chooses excess capacity and the private firm never chooses under capacity. This reinforced the result obtained by Nishimori and Ogawa (2004).[3]

[1] In particular, that each firm chooses excess capacity has long been analyzed as a strategic device for punishment in private oligopolies with respect to both collusion and entry deterrence. For example, using a supergame approach, Davidson and Deneckere (1990) studied collusive equilibria under the capacity choice formulated in Benoit and Krishna (1987). Dixit (1980), Eaton and Ware (1987), Kamien and Schwartz (1972), and Spence (1977) have considered a situation where the accumulation of excess capacity by each firm plays a role in entry deterrence. Moreover, Stewart (1991) considered the strategic entry deterrence problem by analyzing a case in which a profit-maximizing firm attempts to enter an industry in which a monopoly labor-managed firm has already operated. Subsequently, Zhang (1993) and Haruna (1996) studied the behavior of holding excess capacity to deter entry in a labor-managed industry.

[2] The modern game theoretical analysis on mixed oligopoly can be traced back to the paper of DeFraja and Delbono (1989). Subsequently, in a fashion of DeFraja and Delbono (1989) who consider the public firm as a welfare-maximizer and the private firm as a profit-maximizer, many topics in economics have been explored in the context of a mixed oligopoly, the exception being the capacity choice issue. Corneo and Jeanne (1994), Fjell and Heywood (2002), Fjell and Pal (1996), and Pal and White (1998) addressed the international competition with foreign private firms. Bárcena-Ruiz and Garzón (2006), Kato (2006), and Ohori (2006) explored the environmental policy in a mixed oligopolistic industry.

[3] Subsequently, Lu and Poddar (2006) considered a case wherein firms choose capacity scale under the uncertainty

Ogawa (2006) and Bárcena-Ruiz and Garzón (2007) considered a problem similar to those considered by Nishimori and Ogawa (2004) and Lu and Poddar (2005) in the context of quantity competition and price competition, respectively, in a mixed duopoly with differentiated goods.

Since the three papers of Baumol (1958), Simon (1964), and Williamson (1964), the separation of ownership and management in large corporations has challenged the traditional assumption of a firm as a sole profit-maximizing agent. Subsequently, using a modern multi-stage game theoretical approach, Fershtman and Judd (1987), Sklivas (1987), and Vickers (1985) considered a game where in the first stage, an owner — a profit maximizer in each firm — provides a maneger with an the incentive contract that is a linear combination of profits and sales (the so-called $FJSV$ delegation contract), and in the second stage, the managers, knowing both compensation schemes, compete over their quantities in the market.[4] On the other hand, in the context of a mixed oligopoly, the notion of strategic delegation in which the $FJSV$ contract within the public and/or private firm is considered has been extensively analyzed.[5] The seminal paper in this field, Barros (1995) investigated the delegation aspects of the $FJSV$ contracts in principal/agent problems with asymmetric information between an owner and a manager in each firm. Subsequently, White (2001) paid attention to the strategic benefits resulting from managerial incentive contracts of both the public and private firms under complete information.[6] Furthermore, Nakamura and

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of the size of market demand. Moreover, Lu and Poddar (2009) analyzed the endogenous production timing of each firm in the context of a mixed duopoly, taking into consideration all the possible cases of sequential/simultaneous moves regarding the determinations of the levels of each firm's capacity scale and quantity.

[4]The managerial delegation type à la Fershtman and Judd (1987), Sklivas (1987), and Vickers (1985) is referred to as *strategic delegation*. For tractability, in the context of private oligopoly, the $FJSV$ delegation introduced above is frequently applied in several economic situations. González-Maestre and López-Cuñat (2001), Straume (2006), and Ziss (2001) considered the horizontal merger issue, taking into account the $FJSV$ delegation contract within each firm. Kopel and Löffler (2008), Kopel and Riegler (2006), and Zhang and Zhang (1997) considered the influence of the $FJSV$ delegation contract on the equilibrium market outcomes in R&D processes. Collie (1997) and Das (1997) examined the strategic trade policy within each firm in the presence of the $FJSV$ delegation contract.

[5]In particular, in Japan, in the natural gas industries and hospitals and so on, the public firms which coexist and compete with the private firms outsource several businesses to the private sectors. These examples should be analyzed in the context of the strategic delegation, focusing on the separation between ownership and management explicitly. Moreover, as described below in detail, in Japan, the Designated Manager System (Shitei Kanrisha Seido) has been recently introduced as the way of outsourcing of the businesses of the public enterprises in the nursing and hospital service industries.

[6]White (2001) further examined the issue of whether or not to hire managers in both public and private firms in a quantity-setting mixed oligopoly, formulated in Barros (1995), and he showed that private firms hire managers and public firms do not. More recently, Bárcena-Ruiz (2009) considered the same problem as that in White (2001) in a price-setting mixed duopoly with differentiated goods, but his result contrasted with that obtained in White (2001)

Inoue (2007) and Nakamura and Inoue (2009) considered the endogenous timing problem in a mixed duopoly where the separation between ownership and management is observed in each firm in the context of quantity competition and price competition, respectively.[7] Although the above papers considered a situation where owners delegate to managers the decision of selecting strategic variables in the market through $FJSV$ contracts, when multiple strategic variables other than delegation parameters need to be selected within managerial firms, an owner may choose one variable and the corresponding manager may choose another. In reality, not an owner but a manager enforces important managerial decision-making within firms, and consequently, the power of managers differ in every firm. In this paper, in order to take into account the issue of to what degree can owners delegate decision-making to managers within firms in a mixed duopolistic industry, we deal with the capacity choice issue in a mixed duopoly composed of managerial firms. More precisely, we consider a situation wherein either owner or manager chooses the capacity scale in each firm.

The one purpose of this paper is to resolve the question of strategic choice of over or under capacity by the owners of both public and private firms in equilibrium, taking into account the following situations: (*i*) each firm's manager chooses both the capacity scale and output level and (*ii*) each firm's owner selects the capacity scale and the manager chooses the output level. We then consider the following two alternatives as each firm's internal structure: (*a*) *Full delegation* – each firm's owner delegates the decision of the capacity scale and output level to the manager; (*b*) *Partial delegation* – each firm's owner delegates only the output level decision to the manager.[8] There exist some examples in the real mixed oligopolistic industries to validate such a consideration. In the nursing service industry of Japan, health care centers owned by municipalities compete against private ones. Some municipalities are involved in establishing facilities for

where both firms hire managers in equilibrium.

[7]Like other papers on mixed oligopoly with strategic delegation, Saha and Sensarma (2008) considered the mixed Cournot duopolistic industry, focusing on the distributive role of $FVSV$ delegation contracts. Tomaru et al. (2009b) and Tomaru et al. (2009c) addressed the government's optimal subsidization problem when owners and managers enter into $FJSV$ contracts within both public and private firms in the context of a quantity competition and price competition, respectively. In addition, Heywood and Ye (2009) introduced a new type of delegation contract that weighs both profit and welfare like the public firm delegation contract in order to check the robustness of the results obtained by the studies adopting the $FJSV$ contract in mixed oligopolies. In their paper, Heywood and Ye showed that whether the equilibrium welfare increases or decreases with the use of such a delegation contract for public firms depends on the number of private firms and the exact nature of costs.

[8]Cases (*i*) and (*ii*) correspond to those of (*a*) and (*b*), respectively. Moreover, Tomaru et al. (2009a) examined a problem similar to the one considered in this paper. However, Tomaru et al. (2009a) did not take into consideration the possibility of partial delegation.

health care and other tasks such as management and administration are delegated. Such a separation has been proliferating since the introduction of the Designated Manager System (Shitei Kanrisha Seido). This could be regarded as an example of partial delegation. Japan Post is another example. Although fully owned by the government of Japan, all business is delegated to the private manager. However, the Japanese government holds the power to appoint and dismiss the manager. The policy of the Japanese government with respect to Japan Post is a good example of full delegation.

For a description and examination of such competition, we consider a game with the following order of moves: In the first stage, owners decide the delegation parameters of their $FJSV$ contracts. In the second stage, each firm's owner or manager sets the capacity scale through the maximization of her/his objective functions, that is, profit or $FJSV$ contract, and in the third stage, each firm's manager simultaneously sets the quantity level. Thus, in our game, we consider the following four regimes: (1) FF – both the firms' owners choose full delegation; (2) PF – the owner of the public firm chooses partial delegation and that of the private firm chooses full delegation; (3) FP – the public firm's owner chooses full delegation, whereas the private firm's owner chooses partial delegation; and (4) PP – both owners choose partial delegation. Consequently, our results differ from those in the existing literature where the public firm always chooses excess capacity irrespective of its own internal structure and that of the private firm, whereas the private firm whose internal structure is under partial delegation, chooses under capacity. Thus, we find that the results shown in the existing literature where the public firm chooses under capacity and the private firm holds excess capacity are no longer robust with the introduction of the $FJSV$ delegation contract, and along with it, the consideration of manager's power with respect to managerial decision-making within the firms.

The other purpose of this paper is to examine the degree to which owners delegate decision rights to their managers in a mixed duopolistic industry, when each firm chooses multiple strategic variables, *i.e.*, capacity scale and quantity levels in addition with the delegation parameter determined by the owner. Therefore, before begining of the game described above, we consider another game with an additional stage (stage 0) wherein each firm's owner simultaneously selects either full delegation or partial delegation. That is to say, we attempt to analyze the equilibrium internal structure of each firm when considering the separation between ownership and management in the context of the capacity choice problem in a mixed duopoly. Consequently, we obtain the result that in a unique subgame perfect Nash equilibrium, the owner of the public firm chooses partial delegation and that of the private firm chooses full delegation. This result begs the question on the assumption that the internal organization of a public firm is the same as that of a private

firm when managers select strategic variables other than quantity for their firms. Thus, in the equilibrium of capacity choice, the manager's power is stronger in the private firm than in the public firm. In addition, we find that in the equilibrium, the highest social welfare is achieved in the four regimes, whereas the profit of the private firm, *i.e.*, the payoff of the private firm's owner is relatively low.

The remainder of this paper is organized as follows. In Section 2, we formulate the basic model considered in this paper. In Section 3, we present an analysis of the four regimes — FF, PF, FP, and PP — and compare the capacity scales and quantity levels of the public and private firms in each regime. In Section 4, before beginning the first stage of the game considered in section 3, we consider a new game with an additional stage in which each firm's owner simultaneously selects full delegation or partial delegation, and we analyze the equilibrium internal structure of each firm in the context of capacity choice in a mixed duopolistic industry. Section 5 concludes with several remarks. The detailed equilibrium market outcomes in each regime are relegated to the Appendix.

2 Model

We consider a mixed duopolistic market with homogeneous goods. Let us assume that a public firm and private firm are represented by firm 0 and firm 1, respectively; then the inverse demand function is specified as follows:

$$P = a - Q = a - (q_0 + q_1), \quad a \in \mathbb{R}_{++},$$

where P is the market price and q_0 and q_1 denote the quantity level of the public firm and the private firm, respectively.[9]

Each firm employs different technology, which is represented by its quantity level and capacity scale as follows:

$$C_i(q_i, x_i) = m_i q_i + (q_i - x_i)^2, \quad i = 0, 1.$$

This cost function clearly shows the advantage of well-coordinated capacity-quantity choice, which was first presented in Vives (1986). Excess capacity or under capacity would result in inefficiency. Under this U-shaped cost function, the long-run average cost is actually minimized

[9]In the rest of this paper, we assume that a is a sufficiently large real number such that the equilibrium market outcomes in all the regimes are strictly positive.

when quantity is equalized to capacity. In the previous literature on the oligopolies with capacity, production beyond planned capacity is considered to be more costly than production within the capacity-limit chosen before. Despite the fact that idle capacity itself is costly, most studies have neglected the symmetry of costs in the case of excess capacity and under capacity. We explicitly consider such symmetry and use the above formulation of cost functions.

Thus, the profit of each firm is denoted as:

$$\Pi_i = (a - Q)\, q_i - m_i q_i - (q_i - x_i)^2, \quad i = 0, 1,$$

where $m_0 > m_1$ *i.e.*, the public firm is less efficient than the private firm.[10] On the other hand, social welfare is defined as the sum of consumer surplus (denoted by CS) and producer surplus (denoted by PS) *i.e.*,

$$W = CS + PS,$$

where PS is equal to $\Pi_0 + \Pi_1$ and CS is given by

$$CS = \frac{1}{2}Q^2.$$

Following the existing literature on regular mixed oligopolies, the private firm is assumed to be a profit-maximizer, whereas the objective function of the public firm is social welfare. However, we consider a situation wherein the owner of each firm delegates to the manager the decision-making right with respect to quantity level and/or capacity scale.

In all the cases considered in this paper, the owners enter into the $FJSV$ delegation contract with their managers

$$U_i = \Pi_i + \theta_i q_i, \qquad \theta_i \in \mathbb{R}, \quad i = 1, 2,$$

where parameter θ_i measures the relevance of the sales. The manager of firm i can maximize her/his payoff by choosing output q_i that maximizes U_i $(i = 1, 2)$.[11]

Moreover, in this paper, we consider a situation where firms' owners can choose either *partial delegation* or *full privatization*. In partial delegation, each firm's owner her/himself chooses the

[10] Note that if $m_1 \geq m_0$, the profit of firm 1 is equal to zero or negative in the PF and PP cases. In particular, under such an assumption, firm 0's profit is equal to zero or negative.

[11] This can be supported by the assumption that the payoff to the manager of firm i is represented as $\lambda_i + \mu_i V_i$ for some real number λ_i and some positive number μ_i $(i = 1, 2)$. Moreover, following the existing literature in this field, we assume that the payoffs to the managers are negligible as compared to the profits. This reflects the fact that the remuneration to the managers is equalized with the manager's reservation income or opportunity costs, which are constant and exogenous, as indicated in Basu (1995) and White (2001).

capacity scale and each firm's manager decides the level of output. However, in full delegation, both the delegation parameters and output levels are chosen by the firms' owners.

We propose the following three-stage delegation game. In the first stage, the owners of firms decide the parameters of their $FJSV$ delegation contracts, and in the second stage, owners or managers determine their capacity scales from the viewpoint of maximizing of their respective objective functions: the profit or $FJSV$ delegation contract. Finally, in the third stage, the managers of both firms simultaneously decide the levels of their quantities irrespective of the delegation patterns chosen in the first stage.

As described above, we have four possible cases: (FF) the managers in both firms determine their capacity scales and output levels; (PF) in the public firm, the owner and manager choose the capacity scale and quantity level, respectively, and in the private firm, the manager sets both the capacity and output level; (FP) in the public firm, the manager chooses both the capacity scale and quantity level, and in the private firm, the owner and manager decide the capacity scale and output level, respectively; and (PP) in both firms, the owners decide their capacity scales and the managers set their quantities.[12]

3 Equilibrium analysis in the four regimes

In this section, we consider the four regimes. We first analyze the consequence of quantity-setting competition in the fourth and final stage. Since the managers of the public and private firms set their output levels in the third stage, in the FF, PF, FP, and PP cases, the first order condition of firm i is given by

$$\frac{\partial U_i}{\partial q_i} = a - m_i - q_i + 4q_j + 2 + \theta_i = 0, \quad (i = 0, 1;\ i \neq j).$$

This yields

$$q_i\left(x_i, x_j, \theta_i, \theta_j\right) = \frac{1}{15}\left(3a - 4m_i + m_j + 8x_i - 2x_j + 4\theta_i - \theta_j\right), \quad (i, j = 0, 1;\ i \neq j), \tag{1}$$

$$Q\left(x_i, x_j, \theta_i, \theta_j\right) = \frac{1}{5}\left(2a - m_0 - m_1 + 2x_0 + 2x_1 + \theta_0 + \theta_1\right).$$

An increase in one firm raises its output but lowers the rival's output. However, total outputs are increasing with the capacity of each firm. These results are the same as those in a private duopoly.

[12] In our model, we assume that in all four regimes, managers have already been hired in both the public and private firms. Therefore, we do not address the problem of whether or not managers should be hired à la Basu (1995) and Bárcena-Ruiz (2009).

In the following four cases, we conduct analyses of the third and fourth stages by using backward induction, given the output level of each firm presented by (1).

(i) FF

In this case, the managers of both firms decide their capacity scales in the second stage. Thus, in each firm i, the manager's capacity scale is determined through a maximization of the following simplified type of her/his $FJSV$ contract:

$$U_i\left(x_i, x_j, \theta_i, \theta_j\right) = \frac{1}{225}\begin{bmatrix} 18a^2+32m_0^2+2m_1^2+32m_1x_0-97x_0^2-8m_1x_1-64x_0x_1+8x_1^2+16m_1\theta_0 \\ +128x_0\theta_0-32x_1\theta_0+32\theta_0^2-16m_0(m_1+8x_0-2x_1+4\theta_0-\theta_1)-4m1\theta_1 \\ -32x_0\theta_1+8x1\theta_1-16\theta_0\theta_1+2\theta_1^2-12a(4m_0-m_1-8x_0+2x_1-4\theta_0+\theta_1) \end{bmatrix}, \quad (i, j = 0, 1;\ i \neq j).$$

The maximization problems for both firms' managers propose the following reaction functions:

$$x_i = R_i^d(x_j, \theta_0, \theta_1) = \frac{16}{97}(3a - 4m_i + m_j - 2x_j + 4\theta_i - \theta_j), \tag{2}$$

where superscript 'd' denotes the case where one firm's owner delegate the decision of capacity to her/his manager. These reaction functions yield

$$q_i(\theta_0, \theta_1) = \frac{15}{559}(13a - 28m_0 + 15m_1 + 28\theta_0 - 15\theta_1),$$
$$x_i(\theta_0, \theta_1) = \frac{16}{559}(13a - 28m_0 + 15m_1 + 28\theta_0 - 15\theta_1), \quad i = 0, 1.$$

Moreover, we straightforwardly obtain the following result:

$$q_i(\theta_0, \theta_1) - x_i(\theta_0, \theta_1) = -\frac{1}{16}x_i(\theta_0, \theta_1) < 0, \quad i = 0, 1.$$

The above result is summarized as follows:

Lemma 1. *If the owners of both the public and private firms choose "full delegation," i.e., both firms' managers determine the levels of their capacity scale and quantity, both choose excess capacity.*

This lemma states that the over capacities of both firms follow irrespective of managerial delegation parameters θ_0 and θ_1. This result is also indicated by Tomaru et al. (2009a) who addressed a case wherein in a mixed duopoly with differentiated goods, each firm's manager chooses the levels of both the capacity scale and quantity or price.[13]

[13] In a more general setting, with the demand function $p = p(Q)$ and the cost function of each firm $C_i((q_i - x_i), q_i)$, this property still holds as long as the first-order conditions of both firms in the second stage are satisfied ($i = 0, 1$).

In the first stage, the owner of the public firm chooses the delegation parameter of her/his $FJSV$ contract with a view to maximizing social welfare, whereas the owner of the private firm sets her/his delegation parameter such that profit is maximized. The first conditions of both firms are given by

$$\frac{\partial W(\theta_0,\theta_1)}{\partial \theta_0} = \frac{32617a - 194737m_0 + 162120m_1 - 40043\theta_0 - 36345\theta_1}{312481} = 0, \quad \text{and}$$
$$\frac{\partial \Pi_1(\theta_0,\theta_1)}{\partial \theta_1} = \frac{32227a + 37185m_0 - 69412m_1 - 37185\theta_0 - 165368\theta_1}{312481} = 0,$$

respectively. From these equations, we obtain the following equilibrium delegation parameter of each firm:

$$\theta_0^{FF} = \frac{7553699a - 60026399m_0 + 52472700m_1}{9428161}, \quad \theta_1^{FF} = \frac{69412(2a + 225m_0 - 227m_1)}{9428161}$$

Note that the delegation parameter of the public firm is higher than that of the private firm. In fact,

$$\theta_0^{FF} - \theta_1^{FF} = \frac{7414875a - 75644099m_0 + 68229224m_1}{9428161} > 0.$$

Since the owner of the public firm has only one control variable in order to enhance welfare, s/he places more importance on increases in consumer benefits than on cost efficiency improvement through production substitution by changing the capacity of the firm. Thus, the manager is required to act more aggressively than the manager of the private firm.

(ii) PF

In the second stage, the private firm's owner delegates to the manager the decision of quantity as well as capacity, which directly implies that the reaction of the private firm is represented as eq. (2). However, in the public firm, the government selects the capacity scale. Accordingly, capacity is determined such that welfare is maximized as the capacity of the private firm given, which yields the following reaction function:

$$x_0 = R_0^{nd}(x_1,\theta_0,\theta_1) = \frac{54a - 77m_0 + 23m_1 - 46x_1 + 17\theta_0 - 8\theta_1}{71}, \tag{3}$$

where superscript '*nd*' denotes that the owner does not delegate the decision of capacity scale to the manager. From a simple calculation, the owner of the public firm has an incentive to set a larger capacity scale than the manager.

$$R_0^{nd}(x_1,\theta_0,\theta_1) - R_0^{d}(x_1,\theta_0,\theta_1) = \frac{15}{6887}(122a - 195m_0 + 73m_1 - 146x_1 - 193\theta_0 + 24\theta_1) > 0.$$

As pointed out by Nishimori and Ogawa (2004) and Lu and Poddar (2005), the owner of the public firm strategically reduces the firm's capacity scale to induce the more efficient private firm to increase its quantity and capacity scale, which, in effect, leads to enhancing welfare. In contrast to these studies, in this paper, we consider a case wherein both owners delegate to their managers the determination of outputs. The managers attempt to produce less than the welfare-maximizer as long as the delegation parameters are not too large. The owner of the public firm, which cannot control outputs, sets the capacity scale of the public firm at a higher level so as to increase total outputs, even at the cost of deteriorating cost efficiency. Hence, s/he has an incentive to select a higher level of capacity than the manager does.

The reaction functions (2) and (3) yield the following:

$$\begin{cases} x_0(\theta_0, \theta_1) = \frac{1}{361}(202a - 547m_0 + 345m_1 + 159\theta_0 - 248\theta_1), \\ x_1(\theta_0, \theta_1) = \frac{16}{361}(7a + 15m_0 - 22m_1 - 7\theta_0 + 20\theta_1). \end{cases}$$

In the first stage, the owner of the public firm chooses the delegation parameter of her/his $FJSV$ contract such that social welfare is maximized, whereas the owner of the private firm chooses her/his delegation parameter such that profit is maximized. The first order conditions of both the firms are given as follows:

$$\frac{\partial W(\theta_0, \theta_1)}{\partial \theta_0} = \frac{11117a - 62199m_0 + 51082m_1 - 11117\theta_0 - 12537\theta_1}{130321} = 0,$$
$$\frac{\partial \Pi_1(\theta_0, \theta_1)}{\partial \theta_1} = \frac{5(3283a + 7035m_0 - 10318m_1 - 3283\theta_0 - 12280\theta_1)}{130321} = 0.$$

Consequently, we obtain the equilibrium delegation parameters of the $FJSV$ contracts in both firms as follows:

$$\theta_0^{PF} = \frac{264149a - 2360115m_0 + 2095966m_1}{264149}, \quad \theta_1^{PF} = \frac{782292(m_0 - m_1)}{264149}$$

Moreover, each firm's capacity scale and output level is obtained as follows:

$$x_0^{PF} = \frac{264149a - 1977164m_0 + 1713015m_1}{264149}, \quad x_1^{PF} = \frac{1601280(m_0 - m_1)}{264149}, \tag{4}$$
$$q_0^{PF} = \frac{264149a - 2019948m_0 + 1755799m_1}{264149}, \quad q_1^{PF} = \frac{1501200(m_0 - m_1)}{264149}. \tag{5}$$

From eq. (4) to eq. (5), we obtain the following result regarding the level of each firm's output relative to its capacity scale:

$$q_0^{PF} - x_0^{PF} = -\frac{42784(m_0 - m_1)}{264149} = -\frac{2674}{93825}x_0^{PF} < 0, \tag{6}$$
$$q_1^{PF} - x_1^{PF} = -\frac{100080(m_0 - m_1)}{264149} = -\frac{1}{16}x_0^{PF} < 0. \tag{7}$$

These results are summarized as follows:

Lemma 2. *If the public firm chooses "partial delegation" and the private firm chooses "full delegation," i.e., in the public firm, the owner decides the capacity scale and the manager determines the quantity level, and in the private firm, the manager decides the levels of capacity and quantity, both of them choose excess capacity.*

As seen in the case of FF, excess capacity follows in the firm whose manager chooses both quantity and capacity, irrespective of θ_0 and θ_1. Therefore, in the case of PF, the private firm selects excess capacity. Here, all that requires explanation is why the public firm also selects excess capacity. Comparing the public firm's reaction functions R_0^{nd} and R_0^d, we observe that the public firm holds excess capacity when the owner of the firm can determine the capacity level. To mitigate the inefficiency caused by this excess capacity and to encourage the more efficient private firm to produce more, the owner of the public firm adjusts the delegation parameter θ_0. However, the effect of this adjustment is not strong enough to reverse the public firm's excess capacity. This is the intuition behind Lemma 2.

Similar to the case of FF, the public firm's delegation parameter is larger than the private firm's as follows:

$$\theta_0^{PF} - \theta_1^{PF} = \frac{3433147a - 35317027m_0 + 31883880m_1}{4368353} > 0.$$

(iii) FP

In the second stage, the public firm's owner delegates to the manager the decision of quantity as well as capacity, which directly implies that the reaction of the public firm is represented as eq. (2). However, in the private firm, the owner selects the capacity scale, and as a result, the capacity scale is determined such that the private firm's profits are maximized as the capacity of the public firm given, which yields the following reaction function:

$$x_1 = R_1^{nd}(x_0, \theta_0, \theta_1) = \frac{4}{97}(12a + 4m_0 - 16m_1 - 8x_0 - 4\theta_0 + \theta_1),$$

Comparing this reaction function to (2), we have

$$R_1^{nd}(x_0, \theta_0, \theta_1) - R_1^d(x_0, \theta_0, \theta_1) = -\frac{60\theta_1}{97}.$$

This indicates that the owner of the private firm has an incentive to hold a lower capacity than the manager does if s/he proposes the positive delegation contract θ_1. Conversely, the owner

attempts to choose a higher capacity if s/he proposes the negative delegation contract θ_1. The manager's objective function includes $\theta_1 q_1$, which serves as a subsidy from the owner to the manager. Consider that θ_1 is positive. In this case, the manager acts aggressively and thus chooses a higher capacity level to gain a larger market share. However, if θ_1 is negative, which implies that the manager pays tax to the owner, the manager has a weaker incentive to enlarge the market share.

From both firms' reaction functions R_0^d and R_1^{nd}, we have

$$x_0(\theta_0, \theta_1) = \frac{16}{559}(13a - 28m_0 + 15m_1 + 28\theta_0 - 7\theta_1),$$
$$x_1(\theta_0, \theta_1) = \frac{4}{559}(52a + 60m_0 - 112m_1 - 60\theta_0 + 15\theta_1).$$

Subsequently, in the first stage, the owners of both the public and private firms choose their delegation parameters through the maximization of social welfare and profit, respectively.

$$\begin{cases} \frac{\partial W(\theta_0,\theta_1)}{\partial \theta_0} = \frac{32617a - 194737m_0 + 162120m_1 - 40043\theta_0 - 21433\theta_1}{312481} = 0, \\ \frac{\partial \Pi_1(\theta_0,\theta_1)}{\partial \theta_1} = \frac{18915a + 21825m_0 - 40740m_1 - 21825\theta_0 - 72664\theta_1}{312481} = 0. \end{cases}$$

These equations yield the following results:

$$\theta_0^{FP} = \frac{3514627a - 26150527m_0 + 22635900m_1}{4368353}, \quad \theta_1^{FP} = \frac{40740(2a + 225m_0 - 227m_1)}{4368353},$$
$$x_0^{FP} = \frac{240(18441a - 109564m_0 + 91123m_1)}{4368353}, \quad x_1^{FP} = \frac{62608(2a + 225m_0 - 227m_1)}{4368353},$$
$$q_0^{FP} = \frac{225(18441a - 109564m_0 + 91123m_1)}{4368353}, \quad q_1^{FP} = \frac{68880(2a + 225m_1 - 227m_1)}{4368353}.$$

These equilibrium values produce the following results:

$$q_0^{FP} - x_0^{FP} = -\frac{15(18441a - 109564m_0 + 91123m_1)}{4368353} = -\frac{1}{16}x_0^{FP} < 0,$$
$$q_1^{FP} - x_1^{FP} = \frac{6272(2a + 225m_0 - 227m_1)}{4368353} = \frac{56}{559}x_1^{FP} > 0.$$

Thus, we obtain the following result.

Lemma 3. *If the public firm chooses "full delegation" and the private firm chooses "partial delegation," i.e., in the public firm, the manager decides the levels of capacity and quantity, and in the private firm, the owner decides the capacity scale and the manager determines the quantity level, the public firm chooses excess capacity and the private firm chooses under capacity.*

Although the public and private firms tend to choose excess capacity when the two strategic variables — capacity scale and quantity — are determined within both firms in a mixed duopoly, as is described in Tomaru et al. (2009a) and the above analyses, this lemma surprisingly states that the private firm chooses under capacity. This result is even more surprising in comparison with the existing works on mixed duopolies with capacity choice. Nishimori and Ogawa (2004) show that without any managerial delegation, the public firm chooses under capacity and the private firm chooses excess capacity. Lemma 3 indicates that this relationship could be reversed once we introduce managerial delegation into a mixed duopoly.

Now let us explain the intuition behind Lemma 3. Since excess capacity follows in the firm whose manager chooses both quantity and capacity regardless of θ_0 and θ_1, the public firm selects excess capacity in this case (FP). Therefore, we would only like to provide an explanation for the under capacity chosen by the private firm. As stated above, under the positive delegation parameter θ_1, the private firm behaves more aggressively and holds a larger capacity than when the manager controls the capacity scale. Accordingly, the public firm reduces its capacity scale through strategic substitution in the second stage. Furthermore, the negative relationship of reaction function R_0^d and θ_1 intensifies a decrease in x_0. Such a big reduction of x_0 results in a larger market share for the private firm. Therefore, the owner of the private firm attempts to propose a higher delegation parameter. Consequently, a significant decrease in x_0 implies a significant rise in x_1 through strategic substitution, which leads to excess capacity of the private firm.

(iv) PP

In this case, the owners of both firms simultaneously decide their capacity scales in the second stage, and in the third stage, their managers choose the quantity through the maximization of their delegation contracts. Therefore, in the second stage, the capacity scale of each firm is determined by the following two equations:

$$\begin{cases} \frac{\partial W(x_0,x_1;\theta_i,i=0,1)}{\partial x_0} = \frac{2}{225}\left(54a - 77m_0 + 23m_1 - 71x_0 - 46x_1 + 17\theta_0 - 8\theta_1\right) = 0, \\ \frac{\partial \Pi_1(x_0,x_1;\theta_i,i=0,1)}{\partial x_1} = \frac{2}{225}\left(48a + 16m_0 - 64m_1 - 32x_0 - 97x_1 - 16\theta_0 + 4\theta_1\right) = 0. \end{cases}$$

Thus, we obtain the following results:

$$x_0\left(\theta_0,\theta_1\right) = \frac{1}{361}\left(202a - 547m_0 + 345m_1 + 159\theta_0 - 64\theta_1\right),$$
$$x_1\left(\theta_0,\theta_1\right) = \frac{4}{361}\left(28a + 60m_0 - 88m_1 - 28\theta_0 + 9\theta_1\right).$$

In the first stage, the public and private firm owners choose their delegation parameters with a view to maximizing of social welfare and profit, respectively. Thus, the first order condition of this stage is given by

$$\begin{cases} \frac{\partial W(\theta_0,\theta_1)}{\partial \theta_0} = \frac{11117a-62199m_0+51082m_1-11117\theta_0-6857\theta_1}{130321} = 0, \\ \frac{\partial \Pi_1(\theta_0,\theta_1)}{\partial \theta_1} = \frac{6111a+13095m_0-19206m_1-6111\theta_0-30616\theta_1}{130321} = 0, \end{cases}$$

yielding

$$\begin{aligned} \theta_0^{PP} &= \frac{826745a - 5523759m_0 + 4697014m_1}{826745}, \quad \theta_1^{PP} = \frac{1456164\,(m_0 - m_1)}{826745}, \\ x_0^{PP} &= \frac{826745a - 4215036m_0 + 3388291m_1}{826745}, \quad x_1^{PP} = \frac{2408592\,(m_0 - m_1)}{826745}, \\ q_0^{PP} &= \frac{826745a - 4215036m_0 + 3388291m_1}{826745}, \quad q_1^{PP} = \frac{2622096\,(m_0 - m_1)}{826745}. \end{aligned}$$

$$q_0^{PP} - x_0^{PP} = -\frac{271264\,(m_0 - m_1)}{826745} < 0, \quad q_1^{PP} - x_1^{PP} = \frac{213504\,(m_0 - m_1)}{826745} > 0. \tag{8}$$

Lemma 4. *If the owners of both the public and private firms choose "partial delegation," i.e., each firm's owner decides the capacity scale and the quantity level, the public firm chooses excess capacity and the private firm chooses under capacity.*

From this lemma, which is similar to the FP case, we find that the private firm chooses under-capacity. The intuition behind this fact is given as follows: In this case, like in the PF case, since the owner of the public firm chooses the levels of both the delegation parameter and capacity scale, s/he attempts to produce aggressively. Thus, the owner of the public firm sets the higher value of θ_0, and consequently, the value of θ_1 decreases.[14] On the other hand, in the second stage, given the values of both firms' delegation parameters, the capacity scale of the public firm x_0 is the largest and that of the private firm is the smallest in the four regimes because of the aggressivity of the public firm. In addition, from the strategic substitution of each firm's capacity scale regarding the opponent's delegation parameter, the equilibrium capacity scale of the private firm is the lowest among the four regimes; hence it chooses under capacity. On the other hand, the equilibrium capacity scale of the public firm stays relatively high. As a consequence, the public firm chooses excesss capacity.

From Lemmas 1 to 4, we obtain the following result:

[14] As long as the value of a is sufficiently large, the values of θ_0 and θ_1 are the highest and lowest, respectively, as compared to those of the four regimes.

Proposition 1. *In all the four regimes, the public firm chooses excess capacity. On the other hand, the private firm chooses under capacity (over capacity), if its owner selects "partial delegation" ("full delegation") in the first stage.*

In the existing literature in this field, for example Nishimori and Ogawa (2004) and Ogawa (2006), although the public firm and private firm always choose under capacity and excess capacity, respectively, in a mixed duopoly with homogeneous and substitutable goods and without managerial delegation, we find that the public firm always chooses excess capacity, and the private firm chooses under capacity only in the case of partial delegation, when the separation between ownership and management is taken into account.[15] Therefore, in a mixed duopoly, the capacity scale of each firm relative to its quantity strictly depends on (*i*) the type of firms that the market is composed of: entrepreneurial or managerial and (*ii*) the degree of decision rights delegated by the owner of each firm to the managers. Consequently, the results obtained in the existing literature are no longer robust with the introduction of $FJSV$ delegation and along with it, the consideration of managers' powers within the firms.

3.1 Example

In this subsection, we present some numerical results to highlight the impacts of the demand size and the marginal costs on equilibrium delegation parameters and differentials of quantities and capacities. As described in footnote #9, we consider the case where the demand size is sufficient large, in particular, compared to both firms' marginal costs m_0 and m_1.

Table 1 reports the impacts on equilibrium delegation parameters of public and private firms. For any parameters, a, m_0 and m_1, we immediately see that in each regime the equilibrium delegation parameter of the public firm is larger than that of the private firm. As pointed out in the previous subsections, this result is attributed to the aggressive behavior of the public firm's owner. Indeed, the absolute value of θ_0 is much larger relative to that of θ_1 in each regime. Another noteworthy remark from Table 1 is that the public firm's owner tends to raise θ_0 when m_1 becomes large. This is because the public firm's owner attempts to compensate the loss from the inefficient production by the private firm with the consumers' benefits from output expansion through the aggressive behavior of the public firm. The other observation from Table 1 is that an increase in the demand size a leads to higher delegation parameters of the public firm.

[15] In a price-setting mixed duopoly with substitutable goods, the public firm chooses excess capacity and the private firm chooses under capacity in equilibrium, as shown in Bárcena-Ruiz and Garzón (2007), even if the separation between ownership and management in each firm is not considered.

a	m_0	m_1	θ_0^{FF}	θ_0^{PF}	θ_0^{FP}	θ_0^{PP}	θ_1^{FF}	θ_1^{PF}	θ_1^{FP}	θ_1^{PP}
10	0.1	0	7.37518	9.10652	7.44702	9.33187	0.312893	0.296156	0.396362	0.176132
10	0.1	0.05	7.65345	9.50326	7.70611	9.61593	0.229333	0.148078	0.29051	0.0880661
10	0.5	0	4.82849	5.53261	5.05248	6.65933	0.975491	1.48078	1.23572	0.880661
10	0.5	0.05	5.10677	5.92935	5.31157	6.9434	0.89193	1.3327	1.12987	0.792595
10	0.5	0.1	5.38504	6.32609	5.57066	7.22747	0.808369	1.18462	1.02401	0.704529
20	0.1	0	15.387	19.1065	15.4927	19.3319	0.460137	0.296156	0.582886	0.176132
20	1	0.5	12.4397	15.0326	12.6959	16.1593	1.11537	1.48078	1.41291	0.880661
50	1	0	33.6925	41.0652	34.2419	43.3187	2.39271	2.96156	3.03101	1.76132
50	1	0.5	36.4753	45.0326	36.8328	46.1593	1.55711	1.48078	1.97248	0.880661

Table 1: Equilibrium delegation parameters

Next, we check out the impact of a, m_0, and m_1 on differentials between quantities and capacities, which is given in Table 2. Table 2 shows that Proposition 1 holds for any a, m_0, m_1 as long as a is sufficiently large relative to m_0 and m_1. Like Table 1, Table 2 presents some remarkable properties. First, the absolute value of $q_0 - x_0$ exceeds that of $q_1 - x_1$ in each regime. Second, an increase in a is likely to enlarge the absolute value of both $q_0 - x_0$ and $q_1 - x_1$. However, this result does not hold in only PF and PP cases. This is because both firms' differentials of quantities and capacities do not rely on a in these cases, as indicated in equations (6), (7), and (8).

a	m_0	m_1	$q_0^{FF} - x_0^{FF}$	$q_0^{PF} - x_0^{PF}$	$q_0^{FP} - x_0^{FP}$	$q_0^{PP} - x_0^{PP}$	$q_1^{FF} - x_1^{FF}$	$q_1^{PF} - x_1^{PF}$	$q_1^{FP} - x_1^{FP}$	$q_1^{PP} - x_1^{PP}$
10	0.1	0	−0.588572	−0.0161969	−0.595603	−0.0328111	−0.0530114	−0.0378877	0.0610207	0.0258246
10	0.1	0.05	−0.606094	−0.00809846	−0.611248	−0.0164055	−0.0388542	−0.0189439	0.0447246	0.0129123
10	0.5	0	−0.423194	−0.0809846	−0.445115	−0.164055	−0.165271	−0.189439	0.190241	0.129123
10	0.5	0.05	−0.440717	−0.0728861	−0.46076	−0.14765	−0.151114	−0.170495	0.173945	0.116211
10	0.5	0.1	−0.458239	−0.0647877	−0.476405	−0.131244	−0.136957	−0.151551	0.157649	0.103299
20	0.1	0	−1.21849	−0.0161969	−1.22883	−0.0328111	−0.0779579	−0.0378877	0.0897363	0.0258246
20	1	0.5	−1.02161	−0.0809846	−1.04668	−0.164055	−0.18897	−0.189439	0.217521	0.129123
50	1	0	−2.73614	−0.161969	−2.78991	−0.328111	−0.405381	−0.378877	0.466629	0.258246
50	1	0.5	−2.91136	−0.0809846	−2.94635	−0.164055	−0.26381	−0.189439	0.303668	0.129123

Table 2: Differentials of quantities and capacities

4 Extension

In this section, in order to disclose the equilibrium internal structures of both the public and private firms as a subgame perfect Nash equilibrium, we consider a game with an additional stage (stage 0) before beginning the game presented in the previous section. In stage 0, each firm's owner simultaneously chooses partial delegation or full delegation. Before concretely deriving such a subgame perfect Nash equilibrium, we examine the influence of their quantity-capacity relationships on some variables. For this purpose, we compare the equilibrium market outcomes among the four regimes, such as the output, capacity scale, and cost inefficiency of both firms, and the total output and cost inefficiency; subsequently, we conduct a comparison of each firm's objective function in equilibrium in the four regimes, *i.e.*, the equilibrium social welfare and the equilibrium profit of the private firm.

First, we present the rankings of the equilibrium outputs and total costs among the four regimes, summarized as the following lemma.

Lemma 5. *A comparison between the four regimes yields the following rankings of variables.*

(a) $q_0^{PP} > q_0^{PF} > q_0^{FP} > q_0^{FF}, \quad q_1^{FF} > q_1^{FP} > q_1^{PF} > q_1^{PP}, \quad Q^{PP} > Q^{PF} > Q^{FF} > Q^{FP},$

(b) $x_0^{FP} > x_0^{FF} > x_0^{PP} > x_0^{PF}, \quad x_1^{FF} > x_1^{FP} > x_1^{PF} > x_1^{PP},$

(c) $C_0^{FP} > C_0^{FF} > C_0^{PP} > C_0^{PF}, \quad C_1^{FP} > C_1^{FF} > C_1^{PF} > C_1^{PP},$

(d) $C^{FP} > C^{FF} > C^{PP} > C^{PF},$

where $C_i^{jk} = m_i q_i^{jk} + (q_i^{jk} - x_i^{jk})^2$ $(i = 0, 1,\ j, k = P, F)$, *and* $C^{jk} = C_0^{jk} + C_1^{jk}$.

When the owner of the public firm delegates both the quantity and capacity choice to the manager, the output levels are relatively low. In other words, $q_0^{Fi} < q_0^{Pi}$ holds $(i = F, P)$. This is because the control variable to enhance the benefits of consumers is limited to only one, θ_0, for the public firm's owner. Furthermore, the outputs of the private firm increases when the owner of the private firm delegates the choices of the two variables to her/his manager. As stated above, the manager behaves more aggressively than the owner since the manager receives the non-physical subsidy $\theta_1 q_1$. Therefore, we find that $q_1^{iF} > q_1^{iP}$ $(i = F, P)$. The other relationship of each firm's outputs rankings can be explained by strategic substitution. The total output ranking is also explained as a reflection of that of the public firm's outputs since the public firm has a

tendency of excess production and such excess production is likely to dominate the ranking of the private firm's outputs.

Next, let us turn to an exposition of capacity scales. Since the private firm's owner has an incentive to act less aggressive than the manager, the owner attempts to choose a lower capacity level in the case of partial delegation, relative to the case of full delegation; this implies that the private firm can produce the good efficiently. Then, the owner of the public firm tries to control the delegation parameter to replace the output of the public firm with that of the private firm. As a result, $x_0^{iP} > x_0^{iF}$ follows because $q_1(x_0, x_1, \theta_0, \theta_1)$ is decreasing with x_0. In addition, the owner of the public firm selects the capacity scale by taking into account both the firms' cost efficiency when s/he chooses partial delegation; the manager, on the other hand, only takes her/his firm's capacity scale into account. This difference influences whether the capacity is aggressively accumulated and leads to $x_0^{Fi} > x_0^{Pi}$. Thus, we obtain $x_0^{FP} > x_0^{FF} > x_0^{PP} > x_0^{PF}$. The rankings of the private firm's capacity scale are easier to understand. To restrain from drastically deteriorating cost efficiency, the private firm sets its capacity such that the rankings of capacity are consistent with those of outputs.

Finally, we remark about the cost inefficiency C_0^{ij} and C_1^{ij} $(i, j = F, P + \ i \neq \ j)$. As seen in the previous section, the public firm selects excess capacity in all four regimes. This is because accumulated capacity scales are huge regardless of the regime. Thus, the ranking of capacity determines that of C_0^{ij}. Similarly, the private firm's capacity is relatively large in all four regimes, and therefore, capacity ranking strongly reflects that of C_1^{ij}. Nevertheless, cost inefficiency of the public firm is serious, and the ranking of total cost inefficiency C^{ij} completely coincides with that of x_0.

Second, we obtain the results of the equilibrium value rankings of both the objective functions of the public and private firm, *i.e.*, social welfare and profit, as follows:

Lemma 6. *A comparison between the private firm's profits and welfare in the four regimes yields the following rankings.*

(a) $\Pi_1^{FF} > \Pi_1^{FP} > \Pi_1^{PF} > \Pi_1^{PP}$,

(b) $W^{PF} > W^{PP} > W^{FF} > W^{FP}$.

As obtained in the ranking of the equilibrium output presented in Lemma 5, the equilibrium market price decreases when the owner of the public firm selects partial delegation, which leads to fewer profits for the private firm. Indeed, the ranking of price corresponds with that of profits.

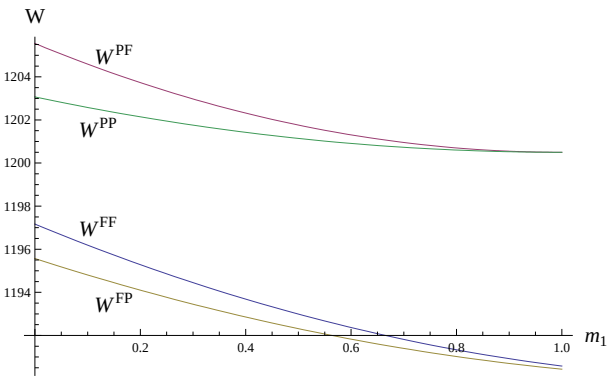

Figure 1: Welfare ranking ($a = 50$ and $m_0 = 1$)

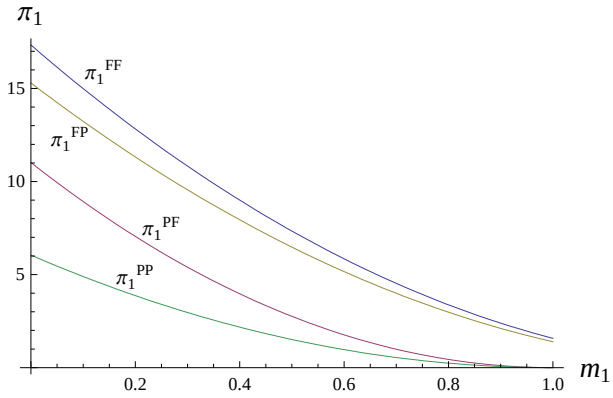

Figure 2: Ranking of private firm's profits ($a = 50$ and $m_0 = 1$)

However, this is not the case when the public firm's owner selects full delegation. This is because cost inefficiency of the private firm rises in the case of FP as compared to the case of FF. Such a rise in cost inefficiency contributes to lowering profit, and thus $\Pi_1^{FP} < \Pi_1^{FF}$. We next explain the intuition behind the ranking of social welfare. As indicated in Lemma 5, when the public owner chooses full delegation, consumer's benefits decrease and social costs increase. Moreover, consumer's benefits in case FF exceed that in case FP ($Q^{FF} > Q^{FP}$), and social costs in FP exceeds that in FF. Therefore, we have $W^{FF} > W^{FP}$. On the other hand, if the public owner selects partial delegation, $W^{PF} > W^{PP}$ since the difference of cost inefficiency outweighs that of consumer's benefits. To understand rankings in Lemma 6 graphically, we present Figures 1 and 2 in which curves of equilibrium welfare and private firm's profits are drawn with a and m_0 fixed. They illustrate that Lemma 6 holds as long as m_1 is not that large as compared to a.

We now discuss the subgame perfect Nash equilibrium in the game with stage 0. We obtain the following result.

Proposition 2. *This game has a unique subgame perfect Nash equilibrium. The owner of the public firm chooses partial delegation, whereas the owner of the private firm chooses full delegation.*

Proof: The statement of this proposition is straightforwardly obtained from Lemma 6. □

This equilibrium is supported by the dominant strategy of each firm's owner. While the owner of the public firm chooses partial delegation irrespective of the strategy of the owner of the private firm, the owner of the private firm always chooses full delegation. If the owner of the public

firm selects partial delegation, both the capacity scale and quantity is determined through the maximization of social welfare, the original objective function of the public firm. Since it is easier for the public firm to control both the private firm's capacity choice and quantity towards the desirable levels from a viewpoint from social welfare, the owner of the public firm always chooses partial delegation. On the other hand, the private firm fundamentally tends to choose full delegation since its market share against that of the public firm is seriously taken into account. In addition, among the four regimes, the highest social welfare is achieved in this equilibrium. Therefore, in equilibrium, the owners of both the public and private firms select desirable roles together from the viewpoint of social welfare. However, in the equilibrium delegation regime, PF, the profit of the private firm is relatively low; hence, the highest social welfare is attained at the expense of the private firm's payoff.

Moreover, from this proposition, we would like to emphasize the equilibrium result where if the owners of both firms select to what degree to delegate decision rights to their managers, the decision of the levels of their capacity scales and quantities and their internal organizations will differ, *i.e.*, the owner of the public firm only delegates the determination of the quantity to the manager, whereas the owner of the private firm delegates the determination of both the capacity scale and quantity level to the manager. Thus, when the separation between ownership and management is explicitly taken into account in the context of capacity choice in a mixed duopoly, the statement of this proposition sheds a striking question to the assumption that the internal organization of the public firm is the same as that of the private firm.

One could ask how our results would be altered if the public firm was privatized. Then, we consider a situation where the owner of firm 0 is a profit-maximizer after privatization, since the shareholdings of firm 0 are bought for the private sector, similar to the case, similar to the case of private firm 1. By conducting the same analysis in this private duopoly as that conducted in the mixed duopoly, we obtain the following result:

Proposition 3. *After privatization of the public firm, the owners of both the privatized and private firms choose full delegation. Furthermore, these firms select excess capacity.*

As is common in the literature on strategic delegation in a private oligopoly, each firm's owner can change her/his strategic position into an advantage situation by manipulating the delegation contract provided to the manager. Therefore, irrespective of the opponent's strategy, each firm's owner chooses the full delegation, and thus, the equilibrium delegation regime, FF, is supported by the dominant strategies of both owners. However, regarding social welfare after privatization of the public firm, we obtain the following negative result:

Proposition 4. *In a mixed duopoly with quantity and capacity choices and managerial delegation, privatization of the public firm decreases social welfare.*

As stated in Proposition 3, in the equilibrium of the private duopoly, both the firms' owners choose full delegation, implying that each firm commits to a large output level with each other. Therefore, the equilibrium social welfare would be expected to increase after privatization rather than before privatization because of the relatively high consumer surplus based on the large total output. However, drawing a comparison between the equilibrium social welfare in the mixed duopoly, W^{PF}, and that in the private duopoly, W^{FF}, we find that privatization of the public firm decreases social welfare with respect to the issue of capacity choice with managerial delegation.

5 Conclusion

This paper examined the capacity choice issue in a mixed duopoly when the separation between ownership and management is observed in each firm. More precisely, we introduced a model where the owner of each firm can select one of the following alternatives: (*i*) *full delegation* – delegating to the manager the decision of capacity scale and quantity level; (*ii*) *partial delegation* – delegating only the quantity setting in the market to the manager. Before conducting an equilibrium analysis of the game, focusing on the capacity scale of each firm relative to its quantity, we considered four regimes: (1) FF – the firms' managers decide both their capacity scales and quantities; (2) PF – in the public firm, the capacity scale is determined by the owner and the output level is selected by the manager, whereas in the private firm, the manager decides the levels of both capacity scale and quantity; (3) FP – in the public firm, the levels of both capacity scale and quantity are determined by the manager, whereas in the private firm, the capacity scale is determined by the owner and the output level is selected by the manager; (4) PP – in both firms, the owners choose their capacity scales and the managers subsequently decide their quantities. Consequently, we obtained the result that the public firm chooses partial delegation irrespective of its own internal structure and that of the private firm, whereas only the private firm whose internal structure only is under partial delegation chooses under capacity; otherwise, excess capacity is chosen. This result is strikingly different from that shown in the existing literature without managerial delegation where in a quantity competition with substitute (including homogeneous) goods, the public firm chooses under capacity as a strategic device, and the private firm tends to choose excess capacity. Thus, in a mixed duopolistic market, whether public and private firms choose excess capacity or under capacity deeply depends on (1) the presence of the separation

between ownership and management and (2) the power of the manager within each firm.

In addition, in order to analyze the endogenous decision-making of each firm's internal structure, we considered a situation where the owners of both firms simultaneously choose partial or full delegation at the beginning of the game, and thus, we derive the subgame perfect Nash equilibrium. Consequently, we found that in a unique subgame perfect Nash equilibrium, the public firm chooses partial delegation and the private firm chooses the full delegation. In the equilibrium, the highest social welfare is achieved among the four regimes, whereas the profit of the private firm, *i.e.*, the payoff of the private firm, is relatively low. Therefore, in the equilibrium, the owners of both firms play socially preferable roles at the expense of the payoff of the private firm's owner. Furthermore, this result sheds a question to the assumption that the decision-rights of the public firm's manager are the same as those of the private firm's manager in the context of the capacity choice problem in a mixed duopolistic industry.

In our future research, we intend to specifically tackle the following two topics. Although in this paper, we assume that the delegation contract in a fashion of Fershtman and Judd (1987), Sklivas (1987), and Vickers (1985) is entered into between the owner and manager in each firm, we should check the robustness of our result against the introduction of other delegation regimes such as *Relative performance* delegation presented in Salas Fumas (1992) and Miller and Pazgal (2001, 2002) and *Market share* delegation in Ritz (2008) and Jansen et al. (2007).[16] Further, as indicated in footnote #11, we do not consider a model where whether a manager is hired within each firm is endogenously determined. Therefore, as our next step, we should address such an issue under the same settings as this paper. Extending this paper in this direction is left for future research.

Appendix

Here, in case FF, we present all the equilibrium market outcomes including each firm's capacity scale and quantity level, and in the other three regimes, we present the equilibrium outcomes other than each firm's delegation parameter, output level, and capacity scale.

[16] More precisely, Salas Fumas (1992) and Miller and Pazgal (2001, 2002) introduced a delegation contract on the basis of a weighted sum of the firm's profit and the rival's profit. Ritz (2008) and Jansen et al. (2007) considered a situation wherein owners provide managers with a delegation contract that is a combination of the firm's own profit and market share.

(i) FF – the regime where the owners of both the firms choose full delegation

In this case, we obtain each firm's equilibrium quantity and capacity scale as follows:

$$x_0^{FF} = \frac{240\,(39593a - 259868m_0 + 220275m_1)}{9428161}, \quad x_1^{FF} = \frac{188160\,(2a + 225m_0 - 227m_1)}{9428161}, \tag{9}$$

$$q_0^{FF} = \frac{225\,(39593a - 259868m_0 + 220275m_1)}{9428161}, \quad q_1^{FF} = \frac{176400\,(2a + 225m_0 - 227m_1)}{9428161}. \tag{10}$$

Taking eqs. (9) and (10) into account, we obtain the following result:

$$\begin{cases} q_0^{FF} - x_0^{FF} = -\frac{15(39593a - 259868m_0 + 220275m_1)}{9428161} = -\frac{1}{16}x_0^{FF} < 0, \\ q_1^{FF} - x_1^{FF} = -\frac{11760(2a + 225m_0 - 227m_1)}{9428161} = -\frac{1}{16}x_1^{FF} < 0. \end{cases}$$

Furthermore, the equilibrium market outcomes are obtained as follows:

$$Q^{FF} = \frac{225\,(41161a - 83468m_0 + 42307m_1)}{9428161}, \quad P^{FF} = \frac{166936a + 225\,(83468m_0 - 42307m_1)}{9428161},$$

$$\Pi_0^{FF} = \frac{225\left(\begin{matrix}5041891399a^2 + 347475822427am_0 - 2497853035076m_0^2 \\ -357559605225am_1 + 4648230247725m_0m_1 - 2145335321250m_1^2\end{matrix}\right)}{88890219841921},$$

$$\Pi_1^{FF} = \frac{14585457600\,(2a + 225m_0 - 227m_1)^2}{88890219841921}, \quad CS^{FF} = \frac{50625\,(41161a - 83468m_0 + 42307m_1)^2}{177780439683842},$$

$$W^{FF} = \frac{225\left(391803659071a^2 - 734396550946am_0 + 3135303930248m_0^2 - 49210767196am_1 - 5536211309550m_0m_1 + 2792711038373m_1^2\right)}{177780439683842}.$$

(ii) PF – the regime where the owner of the public firm chooses partial delegation, whereas the owner of the private firm chooses full delegation

$$Q^{PF} = \frac{264149a - 518748m_0 + 254599m_1}{264149}, \quad P^{PF} = \frac{518748m_0 - 254599m_1}{264149},$$

$$\Pi_0^{PF} = \frac{(m_0 - m_1)\,(67252071251a - 516107211508m_0 + 448855140257m_1)}{69774694201},$$

$$\Pi_1^{PF} = \frac{768728491200\,(m_0 - m_1)^2}{69774694201}, \quad CS^{PF} = \frac{(264149a - 518748m_0 + 254599m_1)^2}{139549388402},$$

$$W^{PF} = \frac{69774694201a^2 - 139549388402am_0 + 774342046888m_0^2 - 1409134705374m_0m_1 + 704567352687m_1^2}{139549388402}$$

(iii) FP – the regime where the owners of the public firm chooses full delegation, whereas the owner of the private firm chooses partial delegation

$$Q^{FP} = \frac{68880\,(2a + 225m_1 - 227m_1)}{4368353}, \quad P^{FP} = \frac{81368a + 9153900m_0 - 4866915m_1}{4368353},$$
$$\Pi_0^{FP} = \frac{225\,(1160436807a^2 + 83376208123am_0 - 536327941604m_0^2 - 85697081737am_1 + 989279675085m_0m_1 - 451791296674m_1^2)}{19082507932609},$$
$$\Pi_1^{FP} = \frac{2762975936\,(2a + 225m_0 - 227m_1)^2}{19082507932609}, \quad CS^{FP} = \frac{225\,(285799a - 610260m_0 + 324461m_1)^2}{38165015865218},$$
$$W^{FP} = \frac{\left[\begin{array}{l}18922540760863a^2 + 122197625008200m_0^2 - 208402636373550m_0m_1 \\ +105127552126213m_1^2 - 2a(17996306821425m_0 + 926233939438m_1)\end{array}\right]}{38165015865218}.$$

(iv) PP – the regime where the owners of both firms choose partial delegation

$$Q^{PP} = \frac{165349a - 318588m_0 + 153239m_1}{165349}, \quad P^{PP} = \frac{318588m_0 - 153239m_1}{165349},$$
$$\Pi_0^{PP} = \frac{(m_0 - m_1)\,(633447885275a - 3303123665716m_0 + 2669675780441m_1)}{683507295025},$$
$$\Pi_1^{PP} = \frac{4131257644224\,(m_0 - m_1)^2}{683507295025}, \quad CS^{PP} = \frac{(165349a - 318588m_0 + 153239m_1)^2}{54680583602},$$
$$W^{PP} = \frac{683507295025a^2 - 1367014590050am_0 + 4193725800616m_0^2 - 7020437011182m_0m_1 + 3510218505591m_1^2}{1367014590050}.$$

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