G-COE GLOPE II Working Paper Series

Mixed Duopoly, Privatization and Subsidization with Excess Taxation Burden

Toshihiro Matsumura and Yoshihiro Tomaru

Working Paper No.19

Mixed Duopoly, Privatization, and Subsidization with Excess Taxation Burden

Toshihiro Matsumura
Institute of Social Science, University of Tokyo
and
Yoshihiro Tomaru*
Graduate School of Economics, Waseda University

March 11, 2009

Abstract

We investigate the optimal subsidy policy in mixed and private oligopolies having excess tax burden. We show that privatization affects both optimal subsidy rate and resulting welfare.

JEL classification: L13, L33, H20

Keywords: Public Enterprise; Mixed Market; Privatization; Optimal Subsidy; Shadow Cost of Public Funding

* *Corresponding author*: Tel.: +81 3 5286 9835. E-mail: y-tomaru@fuji.waseda.jp

1 Introduction

Markets consisting of both private and public firms are called mixed oligopolies, and they retain their importance in many developed, developing, and transitional economies.[1] The pioneering work on mixed oligopoly was performed by Merrill and Schneider (1966). They as well as many subsequent studies assumed that a public firm maximizes welfare (consumers surplus plus firms' profits) while a private firm maximizes its own profits. In their models, governments control the public firms within the market (as an instrument of regulation), instead of using direct policies from outside the markets. However, governments often directly intervene by subsidies for many sectors such as medical care, education, energy, financial, and international trading industries.

Studies on optimal subsidy in mixed oligopolies have gained prominence in recent years. White (1996) showed that the same subsidy rate yields the first-best outcome in both mixed and private oligopolies in his Cournot setting (irrelevant results). Poyago-Theotoky (2001) considered public firms' leadership; Myles (2002) investigated general demand and cost functions; Tomaru (2006) adopted a partial privatization approach formulated by Matsumura (1998); Hashimzade et al. (2007) investigated product differentiation; and Kato and Tomaru (2007) investigated various payoff functions of private firms. These studies demonstrate that the irrelevance result is quite robust.[2]

All these studies neglect the additional cost of public funding: an extra dollar earned in profits has the same social value as an extra dollar in subsidy payments forgone by the government. However, there is an excess tax burden for public funding. We explicitly consider it and then examine the optimal subsidy policy. For this purpose, we apply an approach, which was initially analyzed in Meade (1944) and exploited in Laffont and Tirole (1986). The financial deficit due to subsidy payments is S, and a cost $(1 + \lambda)S$ is incurred.[3]

We analyze the optimal subsidy policy before and after privatization. We find that privatization affects both the optimal subsidy rate and the resulting welfare. After privatization, the optimal subsidy rate either increases or decreases whereas welfare is always reduced.

[1] See, Gil-Moltó and Poyago-Theotoky (2008) and Ishida and Matsushima (2009).

[2] Fjell and Heywood (2004) is an exception. The other works assumed that private firms move simultaneously in private oligopolies, whereas Fjell and Heywood (2004) considered an asymmetric order of moves in private oligopolies. They found that the first-best outcome is not achieved after privatization. In this paper, we do not assume any asymmetry among private firms.

[3] This approach is frequently adopted in contract theory. See, among others, Laffont and Tirole (1993). Capuano and De Feo (2008) introduce this cost in the context of endogenous timing in a mixed oligopoly without subsidy.

2 The Model

The government sets s (the unit subsidy rate). After observing s, firm i ($i = 0, 1$) independently chooses its output q_i. Firm 0 is a welfare-maximizing public firm and firm 1 is a profit-maximizing private firm. They operate in a homogeneous good market with an inverse demand $P = a - Q$, where $Q = q_0 + q_1$. They have an identical cost function $C(q_i) = \frac{1}{2}kq_i^2$ and $k > 0$. Firm i's profit is then

$$\Pi_i = P(Q)q_i - C(q_i) + sq_i = (a - q_i - q_j)q_i - \frac{1}{2}kq_i^2 + sq_i \quad (i, j = 0, 1, \ i \neq j). \tag{1}$$

The government finances the subsidies for the two firms by taxation with excess burden. Social welfare W is given by

$$W(q_0, q_1, \lambda, s) = \int_0^Q P(z)dz - P(Q)Q + \Pi_1(q_1, q_0, s) - (1 + \lambda)\left[sQ - \Pi_0(q_0, q_1, s)\right], \tag{2}$$

where λ represents the unit excess burden. The subsidies payment for the two firms is sQ, but the profits of the public firm would comprise a part of this payment. Thus, the social cost for financing the subsidy payment is given by $(1 + \lambda)(sQ - \Pi_0)$. Moreover, we can rewrite (2) to obtain

$$W(q_0, q_1, \lambda, s) = \left[\int_0^Q P(z)dz - C(q_0) - C(q_1)\right] + \lambda\left[\Pi_0(q_0, q_1, s) - sQ\right] = W(q_0, q_1, 0, 0) + \lambda\left[\Pi_0(q_0, q_1, 0) - sq_1\right].$$

The welfare can be expressed as a weighted average of welfare defined as the net surplus and the public firm's profit after excluding the amount of subsidy for the private firm.

$$W(q_0, q_1, \lambda, s) = \frac{1}{1 + \lambda'}W(q_0, q_1, 0, 0) + \frac{\lambda'}{1 + \lambda'}\left[\Pi_0(q_0, q_1, 0) - sq_1\right].$$

Note that if $s = 0$, this welfare function is the weighted average of welfare as the net surplus and the profit of the public firm. Matsumura (1998) established a model in which a partially privatized firm (owned by both public and private sectors) possesses a similar objective function, since this firm has to respect the interests of both owners. As shown in Matsumura (1998), an increase in the weight on welfare makes a privatized firm become more aggressive. In our setting this property holds, too.

3 Mixed Duopoly

Optimization behaviors by public and private firms yield the following reaction functions:

$$q_0 = R_0(q_1, \lambda) = \frac{(1 + \lambda)(a - q_1)}{1 + k + (k + 2)\lambda}, \quad q_1 = R_1(q_0, s) = \frac{a - q_0 + s}{2 + k}, \tag{3}$$

where R_i is firm i's reaction function. (3) indicates that an increase in s increases the optimal output of firm 1 given q_0, while s does not affect the optimal output of firm 0 given q_1. These properties are common in subsidized mixed oligopolies. A public firm is not interested in income transfer through subsidies because it will eventually return to the owner (public sector). This is why s does not affect the optimal output of firm 0 given q_1. (3) also indicates that an increase in λ decreases the optimal output of firm 0 given q_1, while λ does not affect the optimal output of firm 1 given q_0. As discussed in the previous section, firm 0 cares more about its profit when λ is large. Hence, the optimal output level becomes closer to the profit-maximizing level when λ is large.

Let the superscript 'MS' denote the second-stage equilibrium outcome in a mixed duopoly given s. From (3), we have the following the equilibrium outputs given s:

$$q_0^{MS} = \frac{\{(1+k)a - s\}(1+\lambda)}{(1+3k+k^2)+\lambda(3+4k+k^2)}, \quad q_1^{MS} = \frac{a(k+\lambda+\lambda k)+s(1+k+2\lambda+k\lambda)}{(1+3k+k^2)+\lambda(3+4k+k^2)}. \tag{4}$$

An increase in s directly raises q_1 and it reduces q_0 through strategic interaction. An increase in λ directly reduces q_0 and it increases q_1 through strategic interaction.

We consider the optimal s. The superscript 'M' denotes the subgame perfect Nash equilibrium (SPNE) outcome. Substituting (4) into (2) and then maximizing welfare with respect to s yields

$$s^M = -\frac{a\Omega_0}{\Omega_1}, \tag{5}$$

where $\Omega_0 \equiv k^3\lambda(1+\lambda)^2 + \lambda^2(2+5\lambda) + k^2(-1+2\lambda+9\lambda^2+6\lambda^3) + k(-1-\lambda+9\lambda^2+10\lambda^3)$ and $\Omega_1 \equiv k^3(1+\lambda)^2(1+2\lambda) + \lambda(2+9\lambda+10\lambda^2) + k^2(3+16\lambda+25\lambda^2+12\lambda^3) + k(2+15\lambda+33\lambda^2+21\lambda^3)$.

We can now make two remarks regarding s^M. First, s^M decreases with λ since

$$\frac{ds^M}{d\lambda} = -\frac{a\left\{(\lambda+1)^2k^4 + (9\lambda^2+16\lambda+7)k^3 + (24\lambda^2+36\lambda+13)k^2 + 2(9\lambda^2+8\lambda+1)k + 4\lambda^2\right\}}{\left\{(2\lambda^2+3\lambda+1)k^2 + (8\lambda^2+9\lambda+2)k + 2\lambda(2\lambda+1)\right\}^2} < 0.$$

Second, s^M is not always positive (i.e., the government may impose the production tax). When λ is small, $s^M > 0$. However, when λ is large, s^M is negative. (e.g., if $\lambda \geq 1$, $s^M < 0$.)

Substituting (5) into (4) yields

$$q_0^M = \frac{a(1+\lambda)\left\{\lambda(2+5\lambda) + k^2(1+3\lambda+2\lambda^2) + k(1+7\lambda+7\lambda^2)\right\}}{(1+k+2\lambda+k\lambda)A}, \quad q_1^M = \frac{ak(1+\lambda)^2}{A}, \tag{6}$$

where $A \equiv \lambda(2+5\lambda) + k^2(1+3\lambda+2\lambda^2) + k(2+9\lambda+8\lambda^2)$. The resulting welfare is

$$W^M = \frac{a^2(1+\lambda)^2(k+\lambda+k\lambda)\{2+5\lambda+k(2+3\lambda)\}}{\lambda(2+5\lambda)+k^2(1+3\lambda+2\lambda^2)+k(2+9\lambda+8\lambda^2)}. \tag{7}$$

We compare the output level of the public and private firms.

Lemma 1 *$q_0^M \geq q_1^M$ and the equality holds if and only if $\lambda = 0$.*

Proof From (6), we have

$$q_0^M - q_1^M = \frac{a\lambda(1+\lambda)\left\{2+5\lambda+k^2(1+\lambda)+k(4+5\lambda)\right\}}{(1+k+2\lambda+k\lambda)\left\{\lambda(2+5\lambda)+k^2(1+3\lambda+2\lambda^2)+k(2+9\lambda+8\lambda^2)\right\}} \geq 0,$$

and the equality holds if and only if $\lambda = 0$. **Q.E.D.**

When $\lambda = 0$, the first-best outcome is achieved when $P = C_0' = C_1'$. This implies that $q_0 = q_1$ in the case of the first-best outcome. The government chooses s to induce $P = C_0' = C_1'$. When $\lambda > 0$, this outcome is not optimal. The larger (smaller) output of firm 0 (firm 1) economizes the shadow cost of public funding and s is set to induce such an outcome.

4 Private Duopoly

Next, we consider the private duopoly. Firm 0 is privatized, and then the government chooses s.[4] After observing s, both firms independently choose their outputs. The welfare is given by

$$W(q_0, q_1, \lambda, s) = \int_0^Q P(z)dz - P(Q)Q + \Pi_0(q_1, q_0, s) + \Pi_1(q_1, q_0, s) - (1+\lambda)\left[sQ - V\right], \tag{8}$$

where V is the revenue from selling the stocks of firm 0. Let the superscript 'PS' denote the second stage equilibrium outcome in private duopoly given s. The equilibrium outputs are

$$q_0^{PS} = q_1^{PS} = \frac{a+s}{3+k}. \tag{9}$$

We now consider the optimal s. Let the superscript 'P' denote the SPNE outcome under the private duopoly. Substituting (9) into (8) and maximizing it with respect to s yields

$$s^P = -\frac{a\left\{-1+(3+k)\lambda\right\}}{2+k+6\lambda+2k\lambda}. \tag{10}$$

The resulting output and profits of each firm is

$$q_i^P = \frac{a(1+\lambda)}{2+k+6\lambda+2k\lambda}, \quad \Pi_i^P = \frac{a^2(2+k)(1+\lambda)^2}{2(2+k+6\lambda+2k\lambda)^2}, \qquad i = 0, 1. \tag{11}$$

We assume that the financial market is complete, and hence, $V = \Pi_0^P$.[5] Substituting $V = \Pi_0^P$ and (11) into (8) yields the following welfare:

$$W^P = \frac{a^2(1+\lambda)^2(4+2k+14\lambda+5k\lambda)}{2(2+k+6\lambda+2k\lambda)^2}. \tag{12}$$

[4] If s is determined before privatization, s^P is higher than that presented in (10). In this case, our relevance results still hold.

[5] If the model is a multi-period one, V is equal to the discounted sum of profits, not single period profit. However, in this case other components in W such as consumer surplus, profit of firm 1, and shadow cost of public funding must be discounted sum of them. Thus, all of our results do not change if we consider multi-period as long as the discount rate is common for all players.

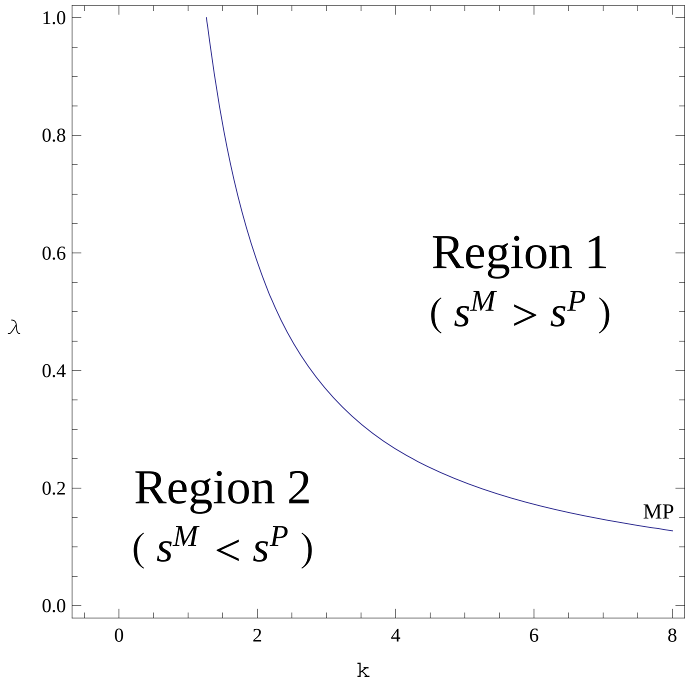

Figure 1: Comparison of the optimal subsidy rates

5 Comparison

First, we compare the optimal subsidy rates. Suppose $\lambda = 0$. From (5) and (10), we have $s^M = s^P$ (irrelevance result). Suppose $\lambda > 0$. In Figure 1, the line MP indicates (λ, k) yielding $s^M = s^P$. In Region 1 (2), $s^M > (<) s^P$. Thus, both $s^M > s^P$ and $s^M < s^P$ is possible. In other words, the optimal subsidy rate can both increase and decrease after privatization. Privatization affects the optimal subsidy rate except for measure zero events. In private duopoly, the government pays the subsidy for both firms; thus, a reduction in s significantly reduces the deadweight loss of public funding. This welfare-improving effect becomes stronger as λ increases. This is why $s^P < s^M$ when λ is large. In contrast, when λ is small, under-production by private firms becomes more serious relative to the direct cost λQ. In private duopoly, an increase in s stimulates production by both firms; thus the government has a stronger incentive for increasing s, and hence, $s^P > s^M$ when λ is small.

Next, we compare welfare. We find that regardless of k and λ, privatization affects welfare.

Proposition 1 *$W^M \geq W^P$ and the equality holds if and only if $\lambda = 0$.*

Proof From (7) and (12), we have

$$W^M - W^P = \frac{a^2\lambda^2(1+\lambda)^2\Psi_1}{\Psi_2(2+k+6\lambda+2k\lambda)^2} \geq 0, \tag{13}$$

where $\Psi_3 \equiv 4+26\lambda+40\lambda^2+k^4(1+3\lambda+2\lambda^2)+k^3(6+21\lambda+16\lambda^2)+k^2(12+51\lambda+47\lambda^2)+k(10+55\lambda+64\lambda^2)>0$ and $\Psi_2 \equiv 2(1+k+2\lambda+k\lambda)\left\{\lambda(2+5\lambda)+k^2(1+3\lambda+2\lambda^2)+k(2+9\lambda+8\lambda^2)\right\}>0$. Hence, we have $W^M \geq W^P$ and the equality holds if and only if $\lambda = 0$. **Q.E.D.**

This result shows that the irrelevance result with welfare does not hold when $\lambda > 0$. This is a sharp contrast to the results in existing literature. Privatization affects welfare because the cost of subsidy is heavier after privatization due to the shadow cost of public funding.

In this paper, we assumed that privatization does not lead to any additional efficiency gain such as reduction in production costs or political gain. If we incorporate these effects, the result that privatization reduces welfare does not hold. However, we believe that the result that privatization is relevant still holds.

References

Capuano, C., De Feo, G., 2008. Mixed duopoly, privatization and the shadow cost of public funds. CORE Discussion Paper 2008019.

Fjell, K., Heywood, J. S., 2004. Mixed oligopoly, subsidization and the order of firm's moves: the relevance of privatization. Economics Letters 83(3), 411–416.

Gil-Moltó, M. J., Poyago-Theotoky, J., 2008. Flexible versus dedicated technology adoption in the presence of a public firm. Southern Economic Journal 74(4), 997–1016.

Hashimzade, N., Khodavaisi, H., Myles, G., 2007. An irrelevance result with differentiated goods. Economics Bulletin 8(2), 1–7.

Ishida, K., Matsushima, N., 2009. Should civil servants be restricted in wage bargaining? A mixed-duopoly approach. Journal of Public Economics, dpi:10.1016.j.jpubeco.2008.11.004.

Kato, K., Tomaru, Y., 2007. Mixed oligopoly, privatization, subsidization and the order of firms' moves: several types of objectives. Economics Letters 96(2), 287–292.

Laffont, J–J., Tirole, J., 1986. Using cost observation to regulate firms. Journal of Political Economy 94(3), 614–641.

Laffont, J–J., Tirole, J., 1993. A theory of incentives in procurement and regulation. Cambridge, MA: MIT press.

Matsumura, T., 1998. Partial privatization in mixed duopoly. Journal of Public Economics 70(3), 473–483.

Meade, J. E., 1944. Price and output policy of state enterprise. Economic Journal 54(215/216), 321–339.

Merrill, W. C., Schneider, N., 1966. Government firms in oligopoly industries: A short-run analysis. Quarterly Journal of Economics 80(3), 400–412.

Myles, G., 2002. Mixed oligopoly, subsidization, and the order of firm's moves: an irrelevance result for the general case. Economics Bulletin 12(1), 1–6.

Poyago-Theotoky, J., 2001. Mixed oligopoly, subsidization, and the order of firm's moves: an irrelevance result. Economics Bulletin 12(3), 1–5.

Tomaru, Y., 2006. Mixed oligopoly, partial privatization and subsidization. Economics Bulletin 12(5), 1–6.

White, M. D., 1996. Mixed oligopoly, privatization and subsidization. Economics Letters 53(2), 189–195.