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Endogenous Timing in a Mixed Duopoly: Price Competition with Managerial Delegation

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Endogenous Timing in a Mixed Duopoly:
Price Competition with Managerial Delegation*

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Abstract

We introduce a managerial delegation contract into the model of Bárcena-Ruiz (2007) and examine its influence on a price-setting mixed duopoly in the context of the endogenous timing problem. We obtain the result that the owners of both firms prefer to set their own prices as late as possible, and thus in equilibrium, the firms choose their prices simultaneously in the latter stage of the game. This is in contrast to the findings of Bárcena-Ruiz (2007) that firms choose prices simultaneously in the former stage.

Keywords: mixed duopoly, managerial delegation, endogenous timing

JEL Classifications: D43, L13, L32

1   Introduction

This study investigates the effects of a managerial delegation contract on the analysis of mixed oligopoly when the timing of the firms' moves is endogenous. An industry in which welfare-maximizing public firms compete with profit-maximizing private firms is usually referred to as a mixed oligopoly. In the real world, despite the worldwide wave of privatization since 1980s, many public firms compete with private firms in private goods markets. The analyses of mixed oligopoly intend to explain the impact of public firms and their privatization on market outcomes. The literature of modern theoretical game analyses on mixed oligopolies can be traced back to a study

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of De Fraja and Delbono (1989) and it has offered logic to explain observed phenomenon in mixed oligopolistic market.

The conventional literature on mixed oligopoly mainly assumes that the order of the firms' moves is exogenous. However, in recent years, there has emerged some literature on mixed oligopoly that endogenizes the timing of the firms' moves. Pal (1998), who has done pioneering work in this field assumed a situation in which firms decided quantities and obtained strikingly different results from those in a corresponding private oligopoly, namely, that the public firm can be the follower in equilibrium. Matsumura (2003) and Lu (2006) also considered the issue of endogenous timing in mixed oligopoly. They extended Pal's (1998) model to investigate the situation in which a domestic public firm competes with foreign private firms. Although most of the literature assumed the quantity-setting game, Bárcena-Ruiz (2007) considered the model where a public firm and a private firm, whose goods are substitutes, choose their prices sequentially or simultaneously. The above-mentioned papers adopt the observable delay game of Hamilton and Slutsky (1990) to determine the endogenous decision of firms' moves, and thus, in the models of the four papers, the firms simultaneously decide in an initial stage to move early or late and then play in subsequent stages, based on these timing decisions.

This study examines how a managerial delegation contract affects the above results. We consider the contract à la Fershtman and Judd (1987) and Sklivas (1987). They explicitly modeled the interaction between internal owner-manager contracting and external strategic market competition, according to which, the firms' owners who aim to maximize their own profits hire managers who do not profit-maximize. Both of Fershtman and Judd and Sklivas considered two-stage duopoly games where, at the first stage, owners choose managerial incentive contracts for managers and in the second stage managers compete in the duopolistic market. The Fershtman-Judd-Sklivas model has become popular and has been applied to competition among private firms.[1] As described later, we model the managerial delegation based on Lambertini (2000). Considering such a delegation contract, we analyze a case in which the public sector indirectly manages its firm by delegating managerial decision-making to an agent, in contrast to the conventional analysis in which all managerial decision-making is directly carried out by the public sector. In this regard, however, there exist a few studies on managerial delegation in mixed oligopoly. Barros (1995) and White (2001) were based on Fershtman and Judd (1987) and Sklivas (1987) regarding the basic structures of the models and assumed that each of the owners provided a linear combination of profit and sales

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[1]Fershtman et al. (1991), Polo and Tedeschi (1992), Bárcena-Ruiz and Paz Espinoza (1996) and Lambertini (2000) also considered the interplay between market competition and the internal organization of the firm in the fashion of Fershtman and Judd (1987) and Sklivas (1987).

revenue as a managerial incentive contract to their managers.[2] However, they considered simultaneous quantity competition as well as most works on mixed oligopoly. Thus, Nakamura and Inoue (2007), our companion study, examined the effects of the managerial delegation through comparisons with Pal (1998), who analyzed the endogenous timing in quantity-setting mixed oligopoly. In that study, we showed that the delegation contract does not affect the equilibrium timing of quantity-settings and proved the robustness of the result in Pal (1998).

In this paper, we provide the analysis on a price-setting mixed duopoly with the managerial delegation contract. As mentioned above, since Bárcena-Ruiz (2007) examined the endogenous timing in a price-setting mixed duopoly, we clarify the role of the delegation contract in mixed oligopoly by comparing our results with Bárcena-Ruiz (2007). Under price competition with substitute goods, the firms have upward-sloping reaction functions unlike the case of strategic substitutes in a quantity setting.

We explicitly recognize separation of ownership from management and analyze the game with the stage at which owners provide their managers with incentives that deviate from profit maximization, unlike a simple observable delay game in the context of a price-setting mixed duopoly in Bárcena-Ruiz (2007). Thus, in our model, each firm's manager decides his or her own price in the market competition. In the literature on private duopoly, Lambertini (2000) adopted the same approach as ours. He first addressed the issue of the timing for deciding strategic variables (prices/outputs) by each firm's manager in the observable delay game between managerial firms. We also take the same approach concerning managerial delegation. Specifically, the owners of firms provide the incentive contract as a linear combination of the profit Π_i and sales q_i to their managers. Each manager sets prices to maximize his or her own payoff in terms of the incentive contract provided by the owner. Our interest lies in whether the result in Bárcena-Ruiz (2007) varies or not by introducing the managerial delegation. The result is striking and surprising. We show that the public and private firm set prices simultaneously in the later stage, since it is a dominant strategy for the two owners to set prices as late as possible. This result is in contrast to that in Bárcena-Ruiz (2007), in which a public firm and a private firm set prices as soon as possible when both of them are entrepreneurial. Our result, as well as that of Bárcena-Ruiz, justifies and explains the reasoning behind the assumption in most of the literature on price-setting mixed oligopoly that all firms simultaneously set their own prices.

The rest of this paper is organized as follows: In Section 2, we formulate the basic setting, and

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[2]Nishimori and Ogawa (2005) also applied the managerial delegation contract to the mixed oligopolistic setting. They extended the intertemporal contract decision in a private duopoly by Bárcena-Ruiz and Paz Espinoza (1996) into the mixed duopolistic case.

in Section 3 we analyze three types of competition of fixed timing and present the equilibrium of our model. Section 4 contains concluding remarks.

2 Setting

We adopt a simplified version of the linear demand model initiated by Dixit (1979) and applied by Singh and Vives (1984). We have an economy composed of a monopolistic sector with a public firm and a private firm, and a competitive sector producing the numeraire good. Each of the firms produces a differentiated good in the monopolistic sector. In the rest of this paper, we often refer to the public firm as Firm 0 (private firm as Firm 1) and the owner of the public firm as Owner 0 (owner of private firm as Owner 1). There exists a continuum of consumers of the same type with a utility function linear and separable in the numeraire good. The representative consumer maximizes $U(q_0, q_1) - p_0 q_0 - p_1 q_1$, where $q_i \geq 0$ is the amount of the good i that Firm i produces and p_i is its price $(i = 0, 1)$. We assume that the function $U(q_0, q_1)$ is quadratic, strictly concave, and symmetric in q_0 and q_1:

$$U(q_0, q_1) = a(q_0 + q_1) - \frac{1}{2}(q_0^2 + 2bq_0q_1 + q_1^2) \qquad a > 0, \quad b \in (0, 1),$$

where b represents the degree of product differentiation. The specification implies the following direct demand function:

$$q_i = \frac{a(1 - b) - p_i + bp_j}{1 - b^2} \qquad i, j = 0, 1; \; i \neq j.$$

Each firm has an identical technology represented by a constant marginal cost function $C(q_i) = cq_i$ $(c \geq 0)$. The profit function of Firm i is denoted by

$$\Pi_i = (p_i - c)q_i \qquad i, j = 0, 1; \; i \neq j,$$

where q_i is given by the above demand function. Social welfare, denoted by W, is measured as the sum of consumer surplus (denoted by CS) and producer surplus (denoted by PS).

$$W = CS + PS,$$

where $PS = \Pi_0 + \Pi_1$ and consumer surplus is given by

$$CS = U(q_0, q_1) - p_0 q_0 - p_1 q_1 = \frac{2a^2(1 - b) + p_0^2 - 2bp_0p_1 + p_1^2 - 2a(1 - b)(p_0 + p_1)}{2(1 - b^2)}.$$

Owner 0 is assumed to be a welfare maximizer, while Owner 1 is assumed to maximize his or her own profit.

In addition, This paper focuses on the managerial aspects of firms. Thus, we consider the situation where the firms' owners decide to delegate control to managers. To formalize managerial delegation, we follow Lambertini (2000). Each of the owners can assess the performance of his or her manager according to two readily observable indicators, *i.e.*, the profit and output of the firm. Following the precedents of the literature on managerial incentive contract, we assume that the manager of Firm i maximizes the following function $V_i(\Pi_i, q_i)$:

$$V_i(\Pi_i, q_i) = \Pi_i + \theta_i q_i \qquad \theta_i \in \mathbb{R}, \quad i = 0, 1,$$

where parameter θ_i identifies the weight attached to the value of sales.[3] In this delegation regime, the manager of Firm i can maximize his or her payoff by choosing the price p_i that maximizes V_i. This can be supported by the assumption that the payoff to the manager of Firm i is represented as $\lambda_i + \mu_i V_i$ for some real number λ_i and some positive number μ_i. This type of delegation scheme functions as a commitment device, since it is common knowledge before the managers compete against each other.[4] Similar to most literature on managerial delegation, we assume that the effect of the managers' payoffs on profits is negligible, since we emphasize the influence of incentive contracts on market outcomes.

We consider the observable delay game in the context of a price-setting mixed duopoly. Note that in this model, a preplay stage is added to the normal mixed duopolistic game. In the game, the firms' owners first announce in which stage their managers will choose their own prices and are committed to the choice before the price competition. After both the owners' announcements, each of the managers sets his or her own price, knowing when the opponent will set his or her own price.

Formally, the game runs as follows: In the first stage, Owner i independently chooses $t_i \in \{3, 4\}$, where t_i indicates the time at which price p_i should be set ($i = 0, 1$). $t_i = 3$ implies that Firm i's manager sets his or her own price in the third stage, and $t_i = 4$ implies that he or she sets his or her own price in the fourth stage. In other words, $t_i = 3$ means that the manager selects an earlier choice of price, and $t_i = 4$ means that the manager selects the later choice of price. At the end of the first stage, the timing-decisions are revealed. In the second stage, Owners 0 and 1 simultaneously set their respective firm's value of θ_i. In the third stage, the manager of Firm i selects his or her own price p_i when Owner i chooses $t_i = 3$. If the corresponding owner chooses $t_i = 4$, the manager does nothing at this stage. At the end of the third stage, the manager of the rival firm j ($\neq i$) that Owner j chooses $t_j = 4$ in the first stage, observes the value of p_i which the

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[3] Even if we replace q_i with p_i in V_i ($i = 0, 1$), our results hold.

[4] Katz (1991) showed general conditions under which unobserved agency contracts can operates as a commitment device.

manager of Firm i has already decided. In the fourth stage, the manager of Firm i selects his or her own price p_i when Owner i chooses $t_i = 4$. At the end of the game, the market opens and each firm sells its own product. We adopt a subgame perfect Nash equilibrium, and thus the game is solved backward.

3 Results for Endogenous Timing

We analyze the subgame perfect Nash equilibrium in the above four-stage game. In this regard, we use a case in which both Owners 0 and 1 set their respective delegation parameters θ_i to zero (no-delegation case) as a benchmark to clarify the role of managerial delegation ($i = 0, 1$).

3.1 *Price-setting*

In the third and fourth stages, the manager of Firm i chooses his or her price to maximize $V_i(\Pi_i, q_i)$. Since there are three subgames — the simultaneous game (Case S), the sequential game with Firm 0 as the leader (Case L), and the sequential game with Firm 0 as the follower (Case F) — we analyze the respective equilibrium. Figure 1 represents the equilibrium points of three cases. In this figure, point $k \in \{S, L, F\}$ is the equilibrium point in Case k, and $R_i(p_j)$ denotes the reaction curve of Firm i ($i, j = 0, 1; \; j \neq i$).

$$R_i(p_j) = \frac{a(1-b) + c + bp_j - \theta_i}{2}.$$

$\overline{V}_i$ expresses the iso-payoff curve of the manager of Firm i. This figure shows that the price of each firm is a strategic complement for the other firm's price and the increase in θ_i shifts the reaction function $R_i(p_j)$ inward.

3.2 *Managerial Delegation*

In the second stage, the owners of both firms simultaneously choose their delegation parameters to maximize their objectives. In analogy with the price-setting stages, we should classify the analyses into Cases S, L, and F.

Case S.   The reaction functions of both owners, $r_0^S(\theta_1)$ and $r_1^S(\theta_0)$, are

$$r_0^S(\theta_1) = \frac{(2 - b - b^2)^2(a - c) + b^3\theta_1}{4 - 3b^2}, \quad r_1^S(\theta_0) = -\frac{b^2[(2 - b - b^2)(a - c) - b\theta_0]}{4(2 - b^2)}.$$
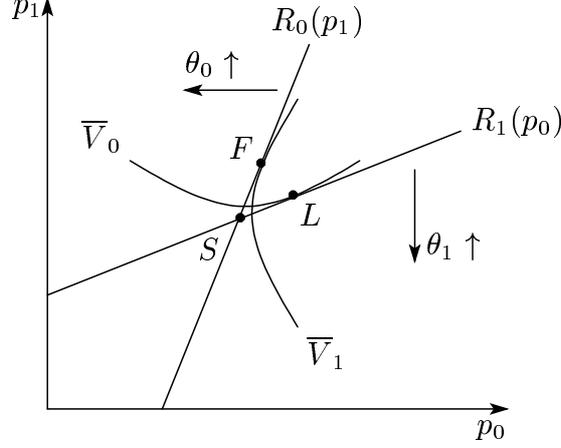
Figure 1: Equilibrium points in the price-setting stages

From these equations, the delegation parameters in equilibrium are as follows:[5]

$$\theta_0^S = \frac{(1-b)(8 - 8b^2 - 2b^3 + b^4)(a-c)}{8 - 8b^2 + b^4}, \quad \theta_1^S = -\frac{b^2(1-b)(2-b^2)(a-c)}{8 - 8b^2 + b^4}.$$

The equilibrium delegation parameter of Owner 1 is negative for any $b \in (0,1)$, whereas that of Owner 0 is positive in the wide range of b ($0 < b < 0.942$). This is because Owner 1 prefers to raise the equilibrium price by shifting his or her reaction function outward for the purpose of increasing profit. On the other hand, since Owner 0 aims at maximizing social welfare, he or she wishes to decrease the price in order to increase consumer surplus. However, when b is sufficiently close to 1 (in other words, the degree of product differentiation is sufficiently low), θ_0^S is negative because, in this case, the equilibrium prices of both firms are close to the marginal cost c and more reduction in price leads to excessive decrease in producer surplus. This mechanism can be explained by comparing it with the no-delegation case as in Figure 2. On the left of this figure, equilibrium point S moves left by increasing θ_0. As a result, Owner 0 can obtain a higher payoff (social welfare).[6] On the other hand, Owner 1 gets a higher payoff (profit) by reducing θ_1.

The respective values in the equilibrium are as follows:

$$p_0^S = \frac{2ab(1-b) + c(8 - 2b - 6b^2 + b^4)}{8 - 8b^2 + b^4}, \quad p_1^S = \frac{2a(2 - 2b - b^2 + b^3) + c(4 + 4b - 6b^2 - 2b^3 + b^4)}{8 - 8b^2 + b^4},$$

$$\Pi_0^S = \frac{2b(1-b)(8 + 2b - 8b^2 - 2b^3 + b^4)(a-c)^2}{(1+b)(8 - 8b^2 + b^4)^2}, \quad \Pi_1^S = \frac{2(1-b)(2-b^2)^3(a-c)^2}{(1+b)(8 - 8b^2 + b^4)^2},$$

$$CS^S = \frac{(40 + 8b - 78b^2 - 18b^3 + 50b^4 + 12b^5 - 12b^6 - 2b^7 + b^8)(a-c)^2}{(1+b)(8 - 8b^2 + b^4)^2},$$

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[5]Henceforth, we indicate the equilibrium values with superscript $k \in \{S, L, F\}$.

[6]The dotted line on the left in Figure 2 represents the reaction function of Firm 0 when its manager maximizes social welfare.
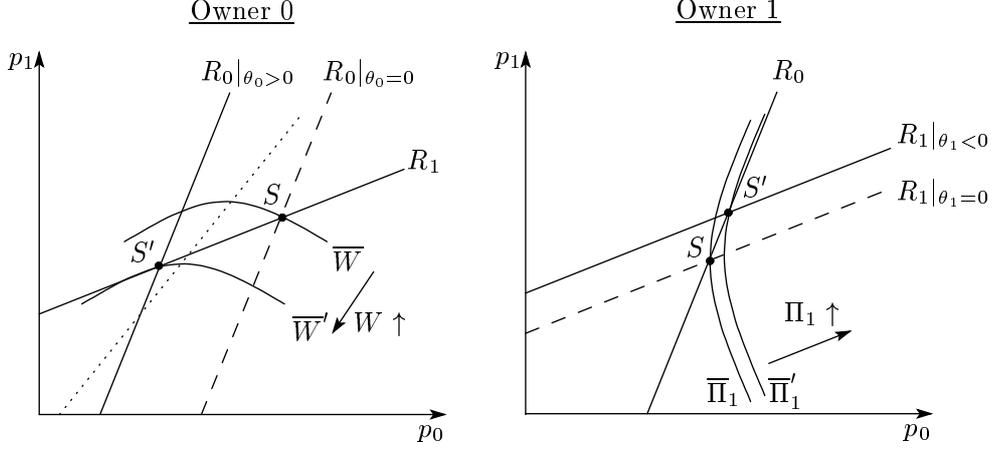
Figure 2: The choice of the delegation parameter in the simultaneous game

$$W^S = \frac{(56 + 8b - 114b^2 - 14b^3 + 74b^4 + 6b^5 - 16b^6 + b^8)(a-c)^2}{(1+b)(8 - 8b^2 + b^4)^2}.$$

Case L.   The reaction functions are

$$r_0^L(\theta_1) = \frac{(8 - 8b - 6b^2 + 5b^3 + b^4)(a-c) + b^3\theta_1}{8 - 10b^2 + 3b^4}$$

$$r_1^L(\theta_0) = -\frac{b^2[(1-b)(4 + 2b - b^2)(a-c) - b(2-b^2)\theta_0]}{(2-b)(2+b)(4 - 3b^2)}.$$

Thus, the equilibrium delegation parameters are as follows:

$$\theta_0^L = \frac{(1-b)(32 - 40b^2 - 4b^3 + 10b^4 + b^5)(a-c)}{(2-b^2)(16 - 20b^2 + 5b^4)}, \quad \theta_1^L = -\frac{2b^2(1-b)(2-b^2)(a-c)}{16 - 20b^2 + 5b^4}.$$

By analogy to Case S, $\theta_1^L < 0$ for any $b \in (0,1)$, but $\theta_0^L > 0$ for $b \in (0, 0.978)$. The reason for this is similar to that in Case S and is illustrated in Figure 3. For both the owners, the change of the delegation parameter from zero (Owner 0 raises θ_0, while Owner 1 reduces θ_1) makes the equilibrium point move from L to L'. If the opponent does not change his or her delegation parameter, each owner gets a higher payoff from the change.

The equilibrium prices, profits, consumer surplus, and social welfare are as follows:

$$p_0^L = \frac{ab(4 - 4b - b^2 + b^3) + c(16 - 4b - 16b^2 + b^3 + b^4)}{16 - 20b^2 + 5b^4},$$

$$p_1^L = \frac{a(8 - 8b - 6b^2 + 6b^3 + b^4 - b^5) + c(8 + 8b - 14b^2 - 6b^3 + 4b^4 + b^5)}{16 - 20b^2 + 5b^4},$$

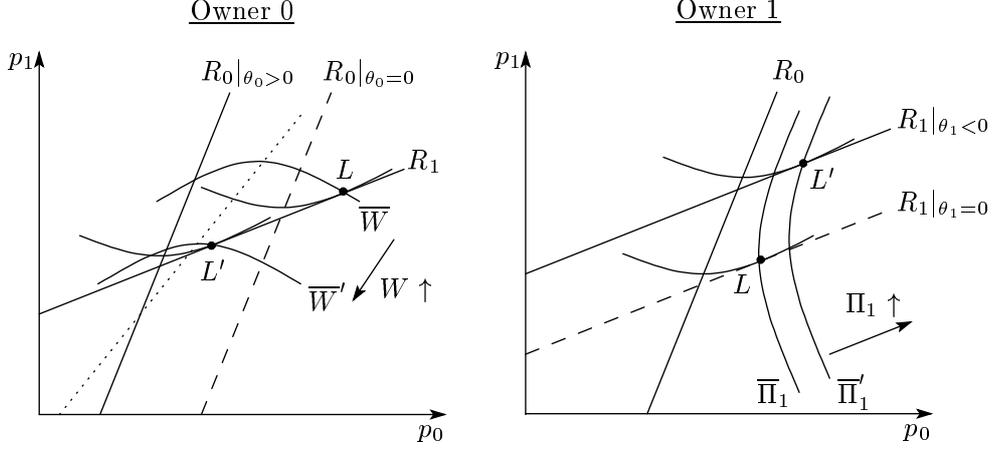$$\Pi_0^L = \frac{b(1-b)(2-b)(2+b)(16 + 4b - 20b^2 - 5b^3 + 5b^4 + b^5)(a-c)^2}{(1+b)(16 - 20b^2 + 5b^4)^2},$$

Figure 3: The choice of the delegation parameter in the sequential game (leader: Firm 0)

$$\Pi_1^L = \frac{(1-b)(2-b)(2+b)(4-3b^2)(2-b^2)^2(a-c)^2}{(1+b)(16-20b^2+5b^4)^2},$$

$$CS^L = \frac{(320+64b-784b^2-176b^3+684b^4+164b^5-251b^6-59b^7+33b^8+7b^9)(a-c)^2}{2(1+b)(16-20b^2+5b^4)^2},$$

$$W^L = \frac{(448+64b-1136b^2-144b^3+1012b^4+108b^5-369b^6-31b^7+47b^8+3b^9)(a-c)^2}{2(1+b)(16-20b^2+5b^4)^2}.$$

Case F. The reaction functions of both owners are

$$r_0^F(\theta_1) = \frac{(16-16b-20b^2+18b^3+7b^4-5b^5)(a-c)+b^3(2-b^2)\theta_1}{16-20b^2+5b^4}, \quad r_1^F(\theta_0) = 0.$$

In this case, Owner 1 sets $\theta_1 = 0$ independent of θ_0. This is because Firm 1 is the Stackelberg leader in the price-setting stages, and thus, the price change by the managerial delegation does not increase its profit. This behavior of Owner 1 (profit maximizer) was pointed out by Lambertini (2000). Therefore, the delegation parameters in the equilibrium of Case F are as follows:

$$\theta_0^F = \frac{(1-b)(2-b)(8+4b-8b^2-5b^3)(a-c)}{16-20b^2+5b^4}, \quad \theta_1^F = 0.$$

As in the above two cases, Owner 0 chooses a positive θ_0 in the wide range of b ($0 < b < 0.961$). Figure 4 depicts this mechanism. For example, even though Owner 1 chooses $\theta_1 < 0$, the profit of Firm 1 does not increase, whereas social welfare is improved by setting $\theta_0 > 0$.

The equilibrium values in Case F are as follows:

$$p_0^F = \frac{2ab(2-2b-b^2+b^3)+c(16-4b-16b^2+2b^3+3b^4)}{16-20b^2+5b^4},$$

$$p_1^F = \frac{a(8-8b-6b^2+6b^3)+c(8+8b-14b^2-6b^3+5b^4)}{16-20b^2+5b^4},$$
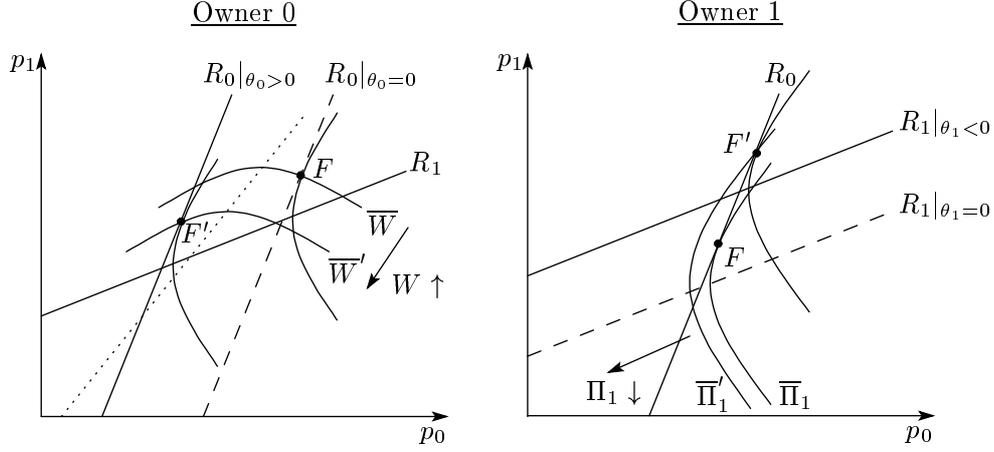
Owner 0   Owner 1

Figure 4: The choice of the delegation parameter in the sequential game (leader: Firm 1)

$$\Pi_0^F = \frac{2b(1-b)(2-b^2)(16+4b-20b^2-4b^3+5b^4)(a-c)^2}{(1+b)(16-20b^2+5b^4)^2},$$

$$\Pi_1^F = \frac{2(1-b)(2-b^2)(4-3b^2)^2(a-c)^2}{(1+b)(16-20b^2+5b^4)^2},$$

$$CS^F = \frac{(10+2b-12b^2-2b^3+3b^4)(a-c)^2}{(1+b)(16-20b^2+5b^4)}, \quad W^F = \frac{(14+2b-18b^2-2b^3+5b^4)(a-c)^2}{(1+b)(16-20b^2+5b^4)}.$$

3.3 Timing Choice

In the first stage, both owners determine whether their firms set their own prices at the third or fourth stage. Matrix 1 represents this situation. Each owner chooses the strategy according to his or her payoff ranking. Thus, we first consider the ranking based on the results of the previous subsection. As a result, we obtain the following lemma.

<center>Owner 1</center>

		$t_1 = 3$	$t_1 = 4$
	$t_0 = 3$	$(W^S,\ \Pi_1^S)$	$(W^L,\ \Pi_1^L)$
Owner 0	$t_0 = 4$	$(W^F,\ \Pi_1^F)$	$(W^S,\ \Pi_1^S)$

Matrix 1: The choice of the timing

Lemma 1. *In the second stage, the equilibrium values of three cases are ranked as follows:*

(i) $\theta_0^L > \theta_0^F > \theta_0^S$ *and* $\theta_1^F > \theta_1^S > \theta_1^L$.

(ii) $p_0^L > p_0^S > p_0^F$ and $p_1^L > p_1^S > p_1^F$.

(iii) $\Pi_0^L > \Pi_0^S > \Pi_0^F$ and $\Pi_1^L > \Pi_1^S > \Pi_1^F$.

(iv) $CS^F > CS^S > CS^L$ and $W^F > W^S > W^L$.

Since in the no-delegation case the prices of both firms in Cases L and F are higher than those in Case S, Owner 0 who aims at maximizing social welfare prefers to reduce the prices in order to increase consumer surplus. Thus, he or she sets the higher delegation parameter in Cases L and F. On the other hand, since Owner 1 can increase the profit of Firm 1 by reducing the delegation parameter from zero in Cases S and L, θ_1^S and θ_1^L are smaller than θ_1^F $(=0)$.

With regard to the equilibrium prices, in Case F, since Firm 1 is the Stackelberg leader of the price-setting stages, Owner 1 cannot increase the profit of Firm 1 by changing the delegation parameter from zero. Hence, the decision of Owner 0 in the second stage is directly reflected in the ensuing behavior of both firms. As a result, p_i^F is the lowest in the three cases $(i = 0, 1)$. On the other hand, in Cases S and L, Owner 1 can increase the profit by reducing the delegation parameter from zero. Considering that $\theta_1^L < \theta_1^S$ and $p_1^L > p_1^S$ at $\theta_1^L = \theta_1^S = 0$, we find that the price of Firm 1 in Case L is higher than that in Case S. In addition, it also raises the price of Firm 0 because in the above price-setting stages, the equilibrium price of Firm 0 is the decreasing function of θ_1. Therefore, p_i^L is the highest in the three cases.

The rank order of the both firms' profits is identical with that of the prices $(\Pi_i^L > \Pi_i^S > \Pi_i^F)$; however, in consumer surplus, the ranking is inverted $(CS^F > CS^S > CS^L)$. Finally, social welfare is ranked as $W^F > W^S > W^L$.

Taking into account Lemma 1, we obtain the following proposition.

Proposition 1. *In the subgame perfect Nash equilibrium of the four-stage game with the managerial delegation, both firms decide their prices simultaneously at the fourth stage ($t_0 = t_1 = 4$).*

By Lemma 1, Owners 0 and 1 wish to set their firms' prices after the price-setting of their opponents. Accordingly, both owners have dominant strategies in the first stage, as described in Matrix 1, to set prices at $t_i = 4$, and thus, the managers of the firms decide their prices simultaneously at the fourth stage.

The equilibrium timing of the price-setting in Bárcena-Ruiz (2007) is different than that in the above proposition. In the former, the owners of both firms wish to set their prices before their opponents do; however, in the latter, they prefer to set them after their opponents because the price-setting of each firm occurs only once in Bárcena-Ruiz (2007), whereas both firms twice set their prices substantially in our model because of managerial delegation. In addition to this, the

fact that in our model, the objective of the public sector to maximize social welfare is reflected in Firm 0 only through the decision of the delegation parameter, also affects the result.

In Bárcena-Ruiz (2007), since the decision for maximizing social welfare is made in the price-setting stages, the equilibrium prices are influenced a great deal by the objective of the Stackelberg leader. Thus, the rank order of the equilibrium prices is $p_i^F > p_i^S > p_i^L$ $(i = 0, 1)$. On the other hand, in our model, since the decisions of both the owners are entered before the price-setting stages, the adjustment of the price through the managerial delegation affects the result. As a result, the ranking is $p_i^L > p_i^S > p_i^F$.

Since a rise in prices increases the profits but decreases consumer surplus, Owner 0 prefers the case in which lower prices are realized. On the other hand, Owner 1 prefers the case in which higher prices are realized. Accordingly, both the owners have dominant strategies to set their prices in the fourth stage.

4   Conclusion

This paper examined a model in which a public firm and a private firm set their own prices sequentially or simultaneously, by focusing on the managerial delegation of firms. We extended Bárcena-Ruiz's (2007) setting as an application of the observable delay game of Hamilton and Slutsky (1990) by introducing a delegation contract à la Fershtman and Judd (1987) and Sklivas (1987). In our model, we showed that in the subgame perfect Nash equilibrium, both the public and the private firms choose their own prices in the later stage. Our result is different from the result in Bárcena-Ruiz (2007).

In a quantity-setting mixed duopoly, Nakamura and Inoue (2007) extended Pal's (1998) observable delay game by introducing managerial incentive contracts in the same manner as ours. They showed that Pal's (1998) results are robust against the introduction of the managerial delegation. In short, they found that two managerial firms sequentially set outputs in equilibrium. However, in a price-setting mixed duopoly, the result of the observable delay game in the context of mixed duopoly varies as a result of introducing the delegation contract. The owners of the public and private firms are reluctant to be the leaders in the case of price competition.

In this paper, we assumed that the technologies of both public and private firms are represented by the same constant marginal cost functions. It can be easily verified that the result that both firms' managers decide prices simultaneously in the fourth stage ($t_0 = t_1 = 4$) remains unchanged against the assumption that the public firm is less efficient than the private firm.

As a valid next step, two extensions of our model can be considered. The first is to generalize

the number of (private) firms in the market and the periods of price competition. We restricted our attention to two firms and two periods in the price competition as in Bárcena-Ruiz (2007). Similar to Pal (1998) and Lu (2006), by considering the more generalized observable delay game in the context of mixed oligopoly, we can discuss the influence of the number of the firms and the degree of product differentiation on the equilibrium outcomes. The other possible extension is to consider the case that some firms are foreign-owned. It seems to more productive to analyze the model in which foreign shareholders are taken into account. Moreover, as is indicated by Lu (2006), one generally considers that foreign firms are more efficient than domestic ones. These issues are left for future research.

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