Wage-rise contract and endogenous timing in international mixed duopoly

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Abstract
The study of Matsumura (2003) investigates a mixed duopoly model, where a domestic public firm and a foreign private firm first choose the timing for choosing their quantities and shows that, in contrast to Pal (1998) discussing a case of domestic competitors, the public firm becomes the leader. This paper examines an international mixed duopoly model, where a domestic public firm competes against a foreign private firm. Each firm first chooses the timing for adopting a wage-rise contract as a strategic commitment. The paper shows the equilibrium of the international mixed model.

Keywords: Wage-rise contract, Endogenous timing, International mixed duopoly, Domestic public firm, Foreign private firm
JEL classification: F23, H42, L30

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1. Introduction

The study of Hamilton and Slutsky (1990) examines the novel issue of endogenous timing in two-payoff-maximizing player games, with important modelling implications for several model in industrial economics. In a preplay stage, players decide whether to select actions in the basic game at the first opportunity or to wait until observing their rivals’ first period actions. In one extended game, players first decide when to select actions without committing to actions in the basic game. The equilibrium has a simultaneous play subgame unless payoffs in a sequential play subgame Pareto dominate those payoffs. In another extended game, deciding to select at the first turn requires committing to an action. They show that both sequential play outcomes are the equilibria only in undominated strategies. Amir (1995) relates to the study of Hamilton and Slutsky (1990) on endogenous timing (with observable delay). He shows, via counterexample, that monotonicity of the best-response functions in a two-player game is not sufficient to derive predictions about the order of moves and this requires, additionally, the monotonicity of each payoff in the other player’s actions. Pal (1998) analyses the subgame perfect Nash equilibrium of a mixed market, where one social-surplus-maximizing public and \( N \geq 1 \) profit-maximizing private firms first choose the timing for selecting their quantities, and shows that the results are strikingly different from those obtained in a corresponding quantity-setting oligopoly with all profit-maximizing firms.\(^1\) Furthermore, Matsumura (2003) examines a Stackelberg mixed duopoly where a social-surplus-maximizing domestic public firm competes against a profit-maximizing foreign private firm and shows that, in contrast to Pal (1998) discussing a case of domestic competitors, the public firm should be the leader.

We study the behaviours of a social-surplus-maximizing domestic public firm and a profit-maximizing foreign private firm in an international mixed duopoly model. Each firm first chooses the timing for adopting the wage-rise-contract policy (henceforth

\(^1\) Following the pioneering work of Merrill and Schneider (1966), the analysis of mixed market models that incorporate social-welfare-maximizing public firms has received increasing attention in recent year. See, for instance, Bös (1986, 2001), Vickers and Yarrow (1988), Cremer, Marchand and Thisse (1989) and Nett (1993) for excellent surveys.
We consider the following situation. In the first stage, each firm independently chooses stage 2 or stage 3. At the end of the first stage, each firm observes the behaviour of the rival. In the second stage, the firm choosing stage 2 adopts WRCP in this stage. At the end of the second stage, each firm observes the behaviour of the rival. In the third stage, the firm choosing stage 3 adopts WRCP in this stage. At the end of the third stage, each firm observes the behaviour of the rival. At the end of the game, each firm independently chooses and sells its actual output. We discuss the equilibrium of the international mixed duopoly model.

The purpose of this paper is to show the role of WRCP as a strategic commitment in the international mixed model where the domestic public firm competes against foreign private firm.

The paper is organized as follows. In Section 2, we formulate the international mixed model. Section 3 discusses the equilibrium of the international mixed model. Section 4 concludes the paper. All proofs of Lemmas and Propositions are presented in the Appendix.

2. The model

Let us consider an international mixed duopoly model with one social-surplus-maximizing domestic public firm (firm 1) and one profit-maximizing foreign private firm (firm 2), producing perfectly substitutable goods. For the remainder of this paper, when $i$ and $j$ are used to refer to firms in an expression, they should be understood to refer to 1 and 2 with $i \neq j$. There is no possibility of entry or exit. The market price is determined by the inverse demand function $p(X)$, where $X = \sum_{i=1}^{2} x_i$ denotes the aggregate quantity. We assume that $p' < 0$ and $p'' < 0$.

The timing of the game runs as follows. In the first stage, each firm independently chooses $t_i \in \{2, 3\}$, where $t_i$ indicates when to adopt WRCP. That is, $t_i = 2$ implies that firm $i$ can adopt WRCP in the second stage, and $t_i = 3$ implies that it can adopt WRCP in the third stage. At the end of the first stage, each firm observes $t_1$ and $t_2$. In the second stage, firm $i$ choosing $t_i = 2$ can adopt WRCP in this stage. At the end of

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2 For details see Ohnishi (2003).
the second stage, each firm observes the behaviour of the rival. In the third stage, firm $i$ choosing $t_i = 3$ can adopt WRCP in this stage. At the end of the third stage, each firm observes the behaviour of the rival. At the end of the game, each firm independently chooses and sells its actual $x_i \geq 0$.

If firm $i$ adopts WRCP, then it chooses an output level $x_i^* \geq 0$ and a wage premium rate $w_i > 0$, and agrees to pay each employee a wage premium uniformly if it actually produces more than $x_i^*$. Hence, firm $i$’s profit is given by

$$
\pi_i = \begin{cases} 
  p(X)x_i - c_i(x_i) & \text{if } x_i \leq x_i^*, \\
  p(X)x_i - c_i(x_i) - (x_i - x_i^*)w_i & \text{if } x_i \geq x_i^*,
\end{cases}
$$

where $c_i(x_i)$ is the cost function. We assume that firms have identical technologies with $c_i' > 0$ and $c_i'' > 0$. The objective of firm 2 is to maximize its own profit.

Domestic social surplus ($S$), which is the sum of consumers’ surplus and firm 1’s profit, is given by

$$
S = \int_0^x p(q) dq - c_i(x_i) - p(X)x_2 \quad \text{if } x_i \leq x_i^*,
$$

$$
\int_0^x p(q) dq - c_i(x_i) - (x_i - x_i^*)w_i - p(X)x_2 \quad \text{if } x_i \geq x_i^*.
$$

The objective of firm 1 is to maximize domestic social surplus.

WRCP specifies a higher marginal cost for output, and firm $i$’s marginal cost exhibits a discontinuity at $x = x_i^*$. In this paper, we will discuss subgame perfect equilibria in undominated strategies.

We derive both firms’ reaction functions in quantities. If firm 1’s marginal cost is $c_i'$, its reaction function is defined by

$$
R_i(x_2) = \arg \max_{[x_i \geq 0]} \left[ \int_0^x p(q) dq - c_i(x_i) - p(X)x_2 \right].
$$

---

3 The assumption of increasing marginal cost is often used in literature studying mixed markets. See, for instance, Harris and Wiens (1980), Ware (1986), Delbono and Scarpa (1995), Fjell and Pal (1996), White (1996), Pal and White (1998), Bárscena-Ruiz and Garzón (2003) and Matsumura and Kanda (2005). If $c_i'' \leq 0$, then firm 1 maximizes social surplus by supplying monopolistically in the market. This assumption is made to eliminate such a trivial solution.

4 This is also used in Hamilton and Slutsky (1990) and Matsumura (2003).
If firm 1’s marginal cost is \( c_1' + w_1 \), its reaction function is defined by

\[
R_1^w(x_2) = \arg \max_{\{x: x > 0\}} \left[ \int_0^x p(q) dq - c_1(x) - (x - x_1^*)w_1 - p(X)x_2 \right].
\] (4)

Therefore, if firm 1 offers WRCP, then its best response is

\[
R_1^w(x_2) = \begin{cases} 
R_1(x_2) \quad &\text{if } x_1 < x_1^*, \\
 x_1^* \quad &\text{if } x_1 = x_1^*, \\
 R_1^w(x_2) \quad &\text{if } x_1 > x_1^*.
\end{cases}
\] (5)

The equilibrium occurs where each firm maximizes its objective with respect to its own output level, given the output level of its rival. Firm 1 aims to maximize social surplus with respect to \( x_1 \), given \( x_2 \). The equilibrium must satisfy the following conditions. If firm 1’s marginal cost is \( c_1' \), the first-order condition is

\[
p - c_1' - p'x_2 = 0.
\] (6)

If firm 1’s marginal cost is \( c_1' + w_1 \), the first-order condition is

\[
p - c_1' - w_1 - p'x_2 = 0.
\] (7)

The second-order condition is

\[
p'' - c_1'' - p''x_2 < 0.
\] (8)

Furthermore, we have

\[
R_1'(x_2) = R_1^w(x_2) = \frac{p''x_2}{p' - p''x_2}.
\] (9)

Thus, \( R_1(x_2) \) and \( R_1^w(x_2) \) are upward sloping.

If firm 2’s marginal cost is \( c_2' \), its reaction function is defined by

\[
R_2(x_1) = \arg \max_{\{x: x > 0\}} \left[ p(X)x_2 - c_2(x_2) \right].
\] (10)

If firm 2’s marginal cost is \( c_2' + w_2 \), its reaction function is defined by

\[
R_2^w(x_1) = \arg \max_{\{x: x > 0\}} \left[ p(X)x_2 - c_2(x_2) - (x_2 - x_2^*)w_2 \right].
\] (11)

Therefore, if firm 2 offers WRCP, then its best response is

\[
R_2^w(x_1) = \begin{cases} 
R_2(x_1) \quad &\text{if } x_2 < x_2^*, \\
 x_2^* \quad &\text{if } x_2 = x_2^*, \\
 R_2^w(x_1) \quad &\text{if } x_2 > x_2^*.
\end{cases}
\] (12)

Firm 2 aims to maximize its profit with respect to \( x_2 \), given \( x_1 \). The equilibrium must satisfy the following conditions. If firm 2’s marginal cost is \( c_2' \), the first-order condition
is
\[ p'x_2 + p - c_2' = 0. \]  
(13)

If its marginal cost is \( c_2' + w_2 \), the first-order condition is
\[ p'x_2 + p - c_2' - w_2 = 0, \]  
(14)

The second-order condition is
\[ 2p' + p''x_2 - c_2'' < 0. \]  
(15)

Furthermore, we have
\[ R'_2(x_i) = R''_2(x_i) = -\frac{p' + p''x_2}{2p' + p''x_2 - c_2''}. \]  
(16)

Thus, \( R'_2(x_i) \) and \( R''_2(x_i) \) are downward sloping.

3. Equilibrium

In this section, we state the equilibrium of the international mixed model. First of all, we will present the next two lemmas.

**Lemma 1:** If firm \( i \) adopts WRCP, then in equilibrium \( x_i = x_i^* \).

**Lemma 2:** Firm \( i \)'s optimal output is smaller when it adopts WRCP than when it does not.

These lemmas provide characterizations of WRCP as a strategic commitment. Lemma 1 means that in equilibrium firm \( i \) does not pay its employees wage premiums. Lemma 2 means that if firm \( i \) offers WRCP, then its optimum output decreases.

Now, we will discuss the following three cases:

Case 1. \( t_1 = 2 \) and \( t_2 = 3 \)
Case 2. \( t_1 = 3 \) and \( t_2 = 2 \)
Case 3. \( t_1 = t_2 = 2 \) \( (t_1 = t_2 = 3) \)

We will discuss these cases in order.
Case 1. $t_1 = 2$ and $t_2 = 3$

In this case, first firm 1 moves, then firm 2 observes firm 1’s move, then firm 2 moves. In the second stage, firm 1 can adopt WRCP. At the end of the second stage, firm 2 observes firm 1’s behaviour. In the third stage, firm 2 can adopt WRCP. At the end of the third stage, firm 1 observes firm 2’s behaviour. At the end of the game, each firm independently chooses its actual output $x_i$, and both social surplus and firm 2’s profit are decided. If unilaterally firm 1 offers WRCP, then its marginal cost increases and thus it decreases its output. Given the output level of firm 2, decreasing firm 1’s output decreases the total market output and raises the market price. Hence, firm 2’s profit increases. Furthermore, firm 2 increases its output because of strategic substitutes. Hence, social surplus may increase. The equilibrium outcome can be stated as follows.

**Proposition 1:** Consider the game where $t_1 = 2$ and $t_2 = 3$. Then there exists an equilibrium which coincides with the Stackelberg solution where firm 1 leads and firm 2 follows. Furthermore, $S^L > S^C$ and $\pi^L_2 > \pi^C_2$. The superscript $L$ denotes the equilibrium outcome where firm 1 is the Stackelberg leader, and the superscript $C$ the equilibrium outcome of the Cournot game without WRCP.

Proposition 1 means that in the second stage, firm 1 chooses $x_{1L}^*$ and $w_{1L}^*$ corresponding to its Stackelberg leader solution and agrees to pay each employee a wage premium uniformly if it actually produces more than $x_{1L}^*$. 

Case 2. $t_1 = 3$ and $t_2 = 2$

In this case, first firm 2 moves. In the second stage, firm 2 can adopt WRCP. At the end of the second stage, firm 1 observes firm 2’s behaviour. In the third stage, firm 1 can adopt WRCP. At the end of the third stage, firm 2 observes firm 1’s behaviour. At the end of the game, each firm independently chooses its actual output $x_i$, and both social surplus and firm 2’s profit are decided. If firm 2 unilaterally offers WRCP, then its marginal cost increases and thus it decreases its output. Given the output level of firm 1, decreasing firm 2’s output decreases the total market output and raises the market price. Furthermore, firm 1 decreases its output because of strategic complements. Given the output level of firm 2, decreasing firm 1’s output decreases the total market output and
raises the market price. Hence, firm 2’s profit increases, while social surplus decreases. The equilibrium outcome can be stated as follows.

**Proposition 2:** Consider the game where $t_1 = 3$ and $t_2 = 2$. Then there exists an equilibrium which coincides with the Stackelberg solution where firm 2 leads and firm 1 follows. Furthermore, $\pi_2^S > \pi_2^C$ and $S^F < S^C$. The superscript $F$ denotes the equilibrium outcome where firm 1 is the Stackelberg follower.

Proposition 2 means that in the second stage, firm 2 chooses $x_2^F$ and $w_2^F$ corresponding to its Stackelberg leader solution and agrees to pay each employee a wage premium uniformly if it actually produces more than $x_2^F$.

Case 3. $t_1 = t_2 = 2$ ($t_1 = t_2 = 3$)

Case 3 is the case in which both firms choose the same stage in the first stage. If each firm chooses $t_i = 2$, this case runs as follows. In the second stage, each firm can independently offer WRCP. At the end of the second stage, each firm observes the behaviour of the rival. In the third stage, neither firm acts. At the end of the game, each firm independently chooses its actual output $x_i$, and both social surplus and firm 2’s profit are decided.

In Matsumura’s (2003) model, if both firms choose the same stage in the first stage, the equilibrium occurs at the Cournot solution. However, in our model, if firm $i$ offers WRCP, then its best response changes as defined by (5) and (12). Hence, even if both firms choose the same stage in the first stage, the equilibrium does not occur at the Cournot solution. If firm 1 unilaterally offers WRCP, then both social surplus and firm 2’s profit increase. Therefore, firm 2 hopes that firm 1 will offer WRCP. On the other hand, if firm 2 unilaterally offers WRCP, then firm 2’s profit increases, while social surplus decreases. Therefore, firm 1 hopes that firm 2 will not offer WRCP. The equilibrium outcome can be stated as follows.

**Proposition 3:** Consider the game where both firms choose the same stage in the first stage. If $\pi_2^L \geq \pi_2^C$, there exists an equilibrium which coincides with the Stackelberg solution where firm 1 is the leader, while if $\pi_2^F > \pi_2^L$, there exists an equilibrium which
coincides with the Stackelberg solution where firm 2 is the leader.

The main result of this study is described by the following proposition.

**Proposition 4:** In the international mixed model, there exist two equilibria: (i) \( t_1 = 2 \) and \( t_2 = 2 \) and (ii) \( t_1 = 2 \) and \( t_2 = 3 \).

4. Conclusion

We have examined an international mixed model, where a social-surplus-maximizing domestic public firm and a profit-maximizing foreign private firm first choose the timing for adopting WRCP. We have shown that there exist two equilibria: (i) ‘stage 2’ for the domestic public firm and ‘stage 2’ for the foreign private firm and (ii) ‘stage 2’ for the domestic public firm and ‘stage 3’ for the foreign private firm. Matsumura (2003) examines a Stackelberg mixed duopoly where a social-surplus-maximizing domestic public firm competes against a profit-maximizing foreign private firm and shows that there exists a unique equilibrium: ‘stage 2’ for the domestic public firm and ‘stage 3’ for the foreign private firm. Our result is different from that of Matsumura (2003). It is thought that WRCP is effective for the foreign private firm.

Appendix

Proof of Lemma 1

First, we prove that if firm 1 adopts WRCP, then in equilibrium \( x_i = x_i^* \). Consider the possibility that \( x_i > x_i^* \) in equilibrium. From (2), social surplus is

\[
S^w = \int_0^X p(q)dq - c_i(x_i) - (x_i - x_i^*)w_i - p(X)x_2.
\]

Here, if \( x_i > x_i^* \), firm 1 must pay its employees wage premiums \( (x_i - x_i^*)w_i \). That is, firm 1 can improve social surplus by rising \( x_i^* \), and the equilibrium point does not change in \( x_i \geq x_i^* \). Hence, \( x_i > x_i^* \) does not result in an equilibrium.

Consider the possibility that \( x_i < x_i^* \) in equilibrium. From (2), firm 1’s marginal cost is
It is impossible for firm 1 to change its output in equilibrium because such a strategy is not credible. That is, if $x_1 < x_1^*$, WRCP does not function as a strategic commitment.

Next, we prove that if firm 2 adopts WRCP, then in equilibrium $x_2 = x_2^*$. Consider the possibility that $x_2 > x_2^*$ in equilibrium. From (1), firm 2’s profit is

$$\pi^w = p(X)x_2 - c_2(x_2) - (x_2 - x_2^*)w_2.$$  

Here, if $x_2 > x_2^*$, firm 2 must pay its employees wage premiums $(x_2 - x_2^*)w_2$. That is, firm 2 can improve its profit by rising $x_2^*$, and the equilibrium point does not change in $x_2 \geq x_2^*$. Hence, $x_2 > x_2^*$ does not result in an equilibrium.

Consider the possibility that $x_2 < x_2^*$ in equilibrium. From (1), firm 2’s marginal cost is $c_2'$. It is impossible for firm 2 to change its output in equilibrium because such a strategy is not credible. That is, if $x_2 < x_2^*$, WRCP does not function as a strategic commitment. Q.E.D.

Proof of Lemma 2

First, firm 1’s social-surplus-maximizing output is smaller when it adopts WRCP than when it does not. From (2), we see that WRCP will never decrease the marginal cost of firm 1. If firm 1’s marginal cost is $c_1'$, the first-order condition is (6), and if its marginal cost is $c_1' + w_1$, the first-order condition is (7). Here, $w_1$ is positive. To satisfy (7), $p - c_1' - p'x_2$ must be positive. Thus, firm 1’s optimum output is smaller when its marginal cost is $c_1' + w_1$ than when its marginal cost is $c_1'$.

Next, firm 2’s profit-maximizing output is smaller when it adopts WRCP than when it does not. From (1), we see that WRCP will never decrease the marginal cost of firm 2. If firm 2’s marginal cost is $c_2'$, the first-order condition is (13), and if its marginal cost is $c_2' + w_2$, the first-order condition is (14). Here, $w_2$ is positive. To satisfy (14), $p'x_2 + p - c_2'$ must be positive. Thus, firm 2’s optimum output is smaller when its marginal cost is $c_2' + w_2$ than when its marginal cost is $c_2'$. Q.E.D.

Proof of Proposition 1

We consider firm 1’s Stackelberg leader output when each firm’s marginal cost is $c_i'$. Firm 1 selects $x_1$, and firm 2 selects $x_2$ after observing $x_1$. If firm 1 is the Stackelberg leader, then it maximizes social surplus $S(x_1, R_2(x_1))$ with respect to $x_1$. Therefore, firm 1’s Stackelberg leader output satisfies the first-order condition:
\[ p - c_1' - p'x_2 - p'x_2R_1' = 0. \]  \hfill (17)

From \( p', R_1' < 0 \), to satisfy \( (17) \), \( p - c_1' - p'x_2 \) must be positive. Hence, firm 1’s Stackelberg leader output is smaller than its Cournot output.

Lemma 2 shows that social-surplus-maximizing output is smaller when firm 1 adopts WRCP than when it does not. From \( (7) \), we see that a decrease in firm 1’s output is decided by the value of \( w_1 \). Let \( w_1 \) be a variable that can take any value more than zero. Thus, the equilibrium coincides with the Stackelberg solution where firm 1 is the leader.

In \( R_2 \), the Stackelberg leader (firm 1) maximizes domestic social surplus and can choose its Cournot output. Hence, we obtain \( S^L \geq S^C \). We show that \( S^L \neq S^C \). Let \( S = \int_0^x p(q)dq - c_1(x_1) - p(X)x_2 \) be continuous and concave with respect to \( x_1 \). In \( R_2 \), social surplus is the highest at firm 1’s Stackelberg leader point, and the further the point on \( R_2 \) gets from firm 1’s Stackelberg leader point, the more social surplus decreases. Firm 1’s Stackelberg leader output is smaller than its Cournot output. Lemma 1 shows that in equilibrium \( x_1 = x_1^* \). Thus, \( S^L > S^C \).

Finally, we show that \( \pi^L_2 > \pi^C_2 \). Firm 1’s Stackelberg leader output is smaller than its Cournot output. Since \( \frac{\partial \pi_2}{\partial x_1} = p'x_2 < 0 \), decreasing \( x_1 \) increases \( \pi_2 \) given \( x_2 \). Firm 2’s optimal strategy must yield at least this profit. Thus, \( \pi^L_2 > \pi^C_2 \). Q.E.D.

Proof of Proposition 2

We consider firm 2’s Stackelberg leader output when each firm’s marginal cost is \( c_i' \). Firm 2 selects \( x_2 \), and firm 1 selects \( x_1 \) after observing \( x_2 \). If firm 2 is the Stackelberg leader, then it maximizes its profit \( \pi_2(x_2, R_1(x_2)) \) with respect to \( x_2 \). Therefore, firm 2’s Stackelberg leader output satisfies the first-order condition:

\[ p - c_2' + p'x_1 + p'x_2R_1' = 0. \]  \hfill (18)

From \( p' < 0 \) and \( R_1' > 0 \), to satisfy \( (18) \), \( p - c_2' + p'x_2 \) must be positive. Hence, firm 2’s Stackelberg leader output is smaller than its Cournot output.

Lemma 2 shows that firm 2’s profit maximizing output is smaller when firm 2 adopts WRCP than when it does not. From \( (14) \), we see that a decrease in firm 2’s output is decided by the value of \( w_2 \). Let \( w_2 \) be a variable that can take any value more than zero. Thus, the equilibrium coincides with the Stackelberg solution where firm 2 is the leader.

In \( R_1 \), the Stackelberg leader (firm 2) maximizes domestic its profit and can choose its
Cournot output. Hence, we obtain $\pi_2^F \geq \pi_2^C$. We show that $\pi_2^F \neq \pi_2^C$. Let $\pi_2 = p(X)x_2 - c_2(x_2)$ be continuous and concave with respect to $x_2$. In $R_1$, firm 2’s profit is the highest at its Stackelberg leader point, and the further the point on $R_1$ gets from firm 2’s Stackelberg leader point, the more firm 2’s profit decreases. Firm 2’s Stackelberg leader output is smaller than its Cournot output. Lemma 1 shows that in equilibrium $x_2 = x_2^*$. Thus, $\pi_2^F > \pi_2^C$.

Finally, we show that $S_F < S_C$. Consider the game where firm 1 is the Stackelberg follower. Since $\partial S(x_2, R_i(x_2)) / \partial x_2 = -p' x_2 > 0$, $S(x_2, R_i(x_2))$ is increasing in $x_2$. Firm 2’s Stackelberg leader output is smaller than its Cournot output. Thus, $S_F = S(x_2^F, R_i(x_2^F)) < S(x_2^C, R_i(x_2^C)) \equiv S_C$. Q.E.D.

Proof of Proposition 3

First, suppose that $\pi_2^L > \pi_2^F$. Then firm 2 hopes that firm 1 will adopt WRCP. Since $S^L > S^F$, firm 1 chooses $x_1^* L$ and $w_1^L$ corresponding to its Stackelberg leader solution and adopts WRCP. Suppose that firm 2 also adopts WRCP. Lemma 1 shows that in equilibrium $x_i = x_i^*$. From (5) and (12), we see that each firm’s reaction function has a zero slope at $x_i = x_i^*$. This implies that even if $x_j$ is increased, $x_i$ is constant. Hence, firm 2 can increase its profit by increasing $x_2$ and $x_2^*$. Firm 2 maximizes its profit by increasing $x_2$ and $x_2^*$ to a point of $R_2$. Thus, the equilibrium coincides with the Stackelberg solution where firm 1 is the leader.

Second, suppose that $\pi_2^L < \pi_2^F$, so that firm 2 chooses $x_2^* F$ and $w_2^F$ corresponding to its Stackelberg leader solution and adopts WRCP. Suppose that firm 1 also adopts WRCP. Lemma 1 shows that in equilibrium $x_i = x_i^*$. From (5) and (12), we see that each firm’s reaction function has a zero slope at $x_i = x_i^*$. This implies that even if $x_j$ is increased, $x_i$ is constant. Hence, firm 1 can increase social surplus by increasing $x_1$ and $x_1^*$. Firm 1 maximizes social welfare by increasing $x_1$ and $x_1^*$ to a point of $R_1$. Thus, the equilibrium coincides with the Stackelberg solution where firm 2 is the leader.

Third, suppose that $\pi_2^L = \pi_2^F$. If firm 2 adopts WRCP, then the equilibrium coincides with the Stackelberg solution where firm 2 is the leader, and social surplus decreases. On the other hand, if firm 1 adopts WRCP, then the equilibrium coincides with the Stackelberg solution where firm 1 is the leader, and social surplus increases. Since $S^L > S^F$, firm 1 chooses $x_1^* L$ and $w_1^L$ corresponding to its Stackelberg leader solution.
and adopts WRCP. Suppose that firm 2 also adopts WRCP. Lemma 1 shows that in equilibrium \( x_i = x_i^* \). From (5) and (12), we see that each firm’s reaction function has a zero slope at \( x_i = x_i^* \). This implies that even if \( x_j \) is increased, \( x_i \) is constant. Hence, firm 2 can increase its profit by increasing \( x_2 \) and \( x_2^* \). Firm 2 maximizes its profit by increasing \( x_2 \) and \( x_2^* \) to a point of \( R_2 \). Thus, the proposition follows. Q.E.D.

Proof of Proposition 4

From Propositions 1-3, we can consider the following two matrices: (i) for \( \pi_2^L < \pi_2^F \),

\[
\begin{array}{c|c|c}
& \text{Stage 2} & \text{Stage 3} \\
\hline
\text{Firm 1} & S^F, \pi_2^F & S^L, \pi_2^L \\
\text{Stage 2} & S^F, \pi_2^F & S^L, \pi_2^L \\
\end{array}
\]

and (ii) for \( \pi_2^L \geq \pi_2^F \),

\[
\begin{array}{c|c|c}
& \text{Stage 2} & \text{Stage 3} \\
\hline
\text{Firm 1} & S^L, \pi_2^L & S^L, \pi_2^L \\
\text{Stage 2} & S^F, \pi_2^F & S^L, \pi_2^L \\
\end{array}
\]

From Propositions 1-2, we see that \( S^F < S^L \). In (i), the equilibrium occurs at ‘stage 2’ for firm 1 and ‘stage 2’ for firm 2. In (ii), the equilibrium occurs at ‘stage 2’ for firm 1 and ‘stage 3’ for firm 2. Q.E.D.

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